Gesture in the Head: Mathematics and Mobility

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Mathematics and gesture. An odd couple: the supreme form of abstract thinking and its crude antithesis, physically expressed feeling. If mathematical symbols engender thought which exceeds that available to speech, gesture seems an atavistic return to a time before speech, allied to simian calls and grunts. Mathematics is everywhere in contemporary digitized culture, gesture is marginal, confined to the cultural periphery of body discipline in sport, military training, and techniques of dance, theatre, and music performance. Certainly, gestures-- cupping hands, jumping on the spot, shrugging, pointing, looping the loop, giving the finger, genuflecting, tying a knot, doing a backflip, spiralling a finger, nodding the head, twisting a wrist, and a thousand other such "disciplined distributions of mobility" (Gilles Chatelet) – appear to show little connection to mathematics or indeed any other intellectual practice.

But gesture was not always regarded as inferior and marginal to thought and language. In the 17th century John Bulwer, pursuing Francis Bacon's dream of finding mankind's original language, the universal tongue that existed before the catastrophe of Babel, homed in on gesture, which Bacon called a "transient heiroglyph", as the key to the search.

Though framing his search as a work of rhetoric, Bulwer, a physician, was interested not in gesture's persuasional features as such, but in its physiological character. He looked to the fact and manner of its embodiment to provide the universalism and aboriginality he sought. One of his essays, *Chirologia*, is devoted to manual gestures.

SLIDE 1 The letters A-S-L finger-spelled in ASL.
CHIROLOGIA: or the NATURALL LANGUAGE of the HAND. Composed of the Speaking Motions and Discoursing Gestures thereof.
Whereunto is added CHIRONOMIA: or The Art of MANUALL RHETORICKE. Consisting of the Naturall Expressions digested by art in the HAND as the chiefest Instrument of Eloquence. (1644)
PATHOMYOTOMIA, or a DISSECTION of the Significative Muscles of the Affections of the Minde (1649)

Bulwer, who created the first finger-spelling alphabet, opens Chirologia with an extraordinary tribute to the hands' abilities to convey meaning and incite affect: "With these hands", he says, "we sue, entreat, beseech, solicit, call, allure, entice, dismiss, grant, deny, reprove, are suppliant, fear, threaten, abhor, repent, pray, instruct, witness, accuse, declare our silence", and so on.

SLIDE 2 *"With these hands ..."*

A fascinating micrososm of mid-17th century English social, religious, and legal encounters.

SLIDE 3 Duplicate Slide 1

In the other essay here, *Panthomyotomia*, Bulwer attempts a metaphorical dissection of the muscles of the face and head so as to reveal their relation to the operations of thought taking place so near to them. In all, Bulwer wrote five books on gesture that constitute him as the (yet to be appreciated) original theoretician of the semiotic body. In the next century others followed, most famously Condillac's attempt to lay out the gestural roots of language, Charles de Brosses' project for a physiological origin for language, and the Abbe de l'Epee's championing of sign language. But by the middle of the 19^{th} centiry the status of bodily gesture had fallen victim to a scientific psychology which subordinated an emotionalized, gesturing body to a rational, thought-producing mind. Perhaps the cruelist consequenceof this subordination of (fluid, essentially female) bodily affect to (hard-edged, essentially masculine) intellect was the fate of the gestural language of the deaf:

SLIDE 4 Anti-gestural prejudice

"Gesture is not the true language of man ... Gesture, instead of addressing the mind, addresses the imagination and the senses. Thus for us, it is an absolute necessity to prohibit that language and to replace it with living speech, the only instrument of human thought."
(Quoted in Harlan Lane, p391)
(On the decision, at the International Conference of Deaf Educators, to enact a total ban on the practice of gestural communication in schools for the deaf. Milan, 1880)

And in the world of the hearing, serious interest in gesture would be confined to constructing various notatation systems and taxonomies of movement that occur in the performing arts.

But this is no longer the case. In the past three decades the importance of gesture in relation to human thought and language has re-emerged from different directions and for several motives within the contemporary scene. And this despite (or rather because of) its 'primitiveness'.

One should not be surprised. Gesture is of the body, and the last three decades have witnessed an explosive focus on the body, amounting to a corporeal upheaval in which, what we have so blithely and simply called 'the body', is being multiply re-configured. Embodiment -- what it means to be/have a body -- is being transformed at many sites, not least the intersection of the bio-medical (gene therapy, cloning, implants), the neurological (brain-scan technologies), and the mediational (the body's encounter with technology -- digital, pharmacological, and virtual). Together they are engendering a new discourse and uncovering hitherto unthought possibilities of human corporeality: "We have only just begun", the philosopher Gilles Deleuze insists, "to understand what the body is capable of". And, one might add, only just begun to discover what gesture is already doing and might yet do.

A recent and radical opening of gesture's horizons was made possible by the tracking and sampling capabilities of digital technology. The writing of speech, as we know, rescues the spoken word from oblivion by transducing it into a text, a fixed and repeatable object of awareness. The result: literate civilization, grammar, dictionaries, and the study of language. What if something less dramatic, but parallel in some sense were possible for gestures? What if gestures could be written down and likewise rescued, brought into consciousness, and examined as discrete, repeatable, free-standing objects? Such a possibility, it transpires, is offered by the techniques of what is known as motion capture. In this, one attaches sensors (responsive to visual, magnetic, or inertial tracking systems) to chosen points on the body (of an animal, a human, a machine) and takes readings -- digitized samples of where in space and real-time these sensors are as the body moves. The result is a representation of the body’s gestures, a digitally recorded trace, which possesses the same mobility, dislocation and freedom from its context of production as alphabetic writing allows to speech. Which is not to say that captured gestures can be assimilated to written texts. Words are signs within a system, according to which they are deciphered, interpreted. Gestures spill outside any matrix of signification that precedes them: they are enacted events that work through their having occurred, their meanings and affects deriving from being performed. (Rotman 2002)

SLIDE 5 optical motion capture -- stills

SLIDE 6 magnetic motion capture -- movie

What is captured is information, sampled data that enables the actual path through three dimensional space of the body being tracked to be re-constructed. Captured gestures already figure in art objects, computer games, animation, and virtual choreography, despite the fact that they lack the means (promised for example by holographic images) to properly project their captured depth.

Relatedly, digitizing gesture, recording it in this way, has opened up the possibility – now being actively developed -- of using a range of hand and head movements as a human-machine interface. So that gesturing to machines would exist alongside the modalities of voice instruction and finger-keyboard interaction; an innovation that, though it complicates the idea, would surely strengthen Bulwer's faith in his chirological project.

A quite different re-understanding of gesture involves a fundamental re-appraisal of its relation to language. Starting with the recognition that the various gestural systems – generically *Sign* -- used throughout the world by the deaf to commune with each other are not, as had long been assumed, imitative, pantomime-like practices, but full-blown (visual) languages, on an semantic, syntactic, and pragmatic par with the (auditory) languages of human speech, and in some respects, in their use of spatially indicated pronouns for example, superior to them. Consequently, ideas of 'language' are in flux, and there is a re-emergence of theories arguing for the gestural origins of language.

At the same time, there are claims for the active presence of gestural modes of semiosis within language use. Thus:

SLIDE 7 *"Considered jointly with speech, gestures open a 'window' onto the mind. ... [T]aking gesture into account, we see patterns not revealed by speech alone and see more comprehensively how* *meanings are constructed. Gesture is not only a display of meaning but is part of the act of constructing meaning itself, adding a 'material carrier' that helps bring meaning into existence ..."* David McNeill Hand and Mind (1992)

Gesture here means gesticulation: the fleeting, idiosyncratic, and seemingly meaningless movements of the hands and arms that accompany everyday speech. His research reveals gestures as deeply embroiled in the narratives they accompany, being intricately folded into the linguistic form and thought-content of what's spoken. McNeill measured the microsecond co-ordination between gestures and individual words or phrases, and observed how gestures concretize spoken abstractions as well as performing metalingual and discourse-pragmatic functions. As a result he hypothesized that gestures and words have a common antecedent. And that they unfold along a dialectic of opposed modes of representation: gestural (imagistic, holistic) and verbal (segmental, analytic) combining sequences of pre-articulated words and simultaneously presented mental images. A hypothesis that suggests that spoken thoughts might start life as yet-to-be-realized gestures.

Gesture research has mushroomed in the last 20 years and McNeill's work on speech is now part of a larger scene of investigation of gesture's role, in particular in connection to mathematical thought. Recent studies include the role of body metaphors in the origin of mathematical concepts, the function of gesture in learning to count, gestural participation in the conceptual planning of talk, the role of gesture in mathematics learning, and the relation of gesture to scientific language and visual representation. One theme that emerges from these studies involves an undoing of separating boundaries, a recognition of interconnectedness. What had been long treated as distinct areas of study of autonomous behaviours – language, vision, motor activity, reasoning – are seen as interconnected and co-productive; an insight that emerges often as a result of new investigative techniques such as digital tracking technology. Let me focus here on one such example.

SLIDE 8 New Avenues for the Microanalysis of Mathematics Learning: Connecting Talk, Gesture, and Eye Motion (Nemirovsky & Ferrara, 2004)
"Thinking encompasses parallel streams of bodily activity (gesturing, talking, walking, etc) which sometimes converge. ... Thinking is not a process that takes place 'behind' or 'underneath' bodily activity, but is the bodily activity itself."

By microanalysis here is meant the real-time (over seconds and minutes), moment-to-moment tracking of the hand, eye, and speech movements of a group of students grappling with a new piece of mathematics. What it reveals is that mathematics learning is a complex and dynamic mix of talking, gesturing, and looking evident for example in what the authors describe as "co-ordinated activity among hands, eyes, and talk in the process of expanding, or bringing into the open, aspects of visual meaning". Generally, they propose that the development of "Children's thinking ... is more akin to an ecology of ideas, co-existing and competing with each other for use, than like monolithic changes from one stage of understanding to the next." (Seigler 1996). An ecology implies an assemblage of different processes and activities sharing sufficient structure or functions to interact with each other in an integrated whole. An essential ingredient here is the activity of the eye.

SLIDE 9 Scanning eye movements of a subject looking at a bust of Nefertiti.

Tracking of eye movements reveals that visual perception, far from being a purely receptive recording (camera snapshot), is in reality an interrogation of a visual scene. Plainly, on a physiological level of disciplined and purposive mobility, seeing, no less than talking, consists of a series of gestures. What is of interest here is the connection, the "co-ordinated activity", which links what is seen with what is spoken. How, in the case of mathematics, does this come about?

If we conceive of mathematics in terms of logical narratives -- chains of implications, calculations, demonstrations told in symbols -- then one possible response can be modelled on the situation revealed by McNeill for speech. According to this, gestures of the eye, the tongue, and the hands might be folded into and co-ordinated with the story being told in symbols by virtue of being generated in parallel with it. This would suggest that the eye/tongue/hand activities be understood as a form of mathematical gesticulation and – as with speech -- would be handling iconic, metalingual and discourse-pragmatic dimensions of the ongoing narrative.

But mathematical thinking also consists of ideas, patterns, relations between concepts, and imagined constructs, as much as it does chains of reasoning. And it is this aspect of thought that determines how the "parallel streams of bodily activity (gesturing, talking, walking, etc) which sometimes converge" can, in converging, constitute an ecology of thought. To say more I need to introduce two neurological items.

The first is the principle of *motor equivalence*, what neuroscientist A. Berthoz calls the "simple and remarkable property of the brain which allows one to perform the same bodily movement with very different effector systems." (1997) Thus a single motor program, such as the one generating the gesture of signing one's name, is impervious to how it is manifested: we can sign with our hand, as we normally do, but also with our elbow, our head, left foot, our eyes, or -- the largest effector system of all -- we can walk our signature in the sand.

The second item I need to introduce comes from a neuroscientific discovery in the last decade. Its source is our monkey here.

SLIDE 10 Mirror Neurons: The Monkey See Monkey 'Do' phenomenon. *"Mirror neurons will do for psychology what DNA did for biology: they will provide a unifying framework and helpl explain a host of mental abilities that have hitherto remained mysterious and inaccesible to experiments."* V.S. Ramachandrian 2004

Neurophysiologist Giaccamo Rizzollati records signals from the premotor area of the frontal lobes of macaque monkeys, in order to identify which individual cells fire when a monkey performs specific actions such as pushing, pulling, picking up an object, putting paw to mouth, etc. These cells appear to be the standard command neurons whose job is to make muscles execute the actions in question. But it turns out they do something more. They also fire when a monkey observes another monkey performing the actions (or a human – which is how in fact they were first noticed). A stunning discovery that is already opening up what was hitherto mysterious.

Humans also possess these mirror neurons, and the implications for the study of the 'human' – for language learning, for an evolutionary account of speech, for imitative behaviour, and for the deep-lying presence of empathy and fellow-feeling – are, if anything, even more momentous than Ramachandran's DNA analogy suggests, since our possession of them has ethical and philosophical implications outside the confines of scientific psychology.

Certainly, if observing someone performing an action causes the corresponding nueuron of your own to fire, then you have the means for linking their affect – insofar as their bodies display it -- to your own, as well as having access, through reading and anticipating their intentions, to their minds. Or, again, consider pure imitation, a fundamental but hitherto deeply puzzling, indeed mysterious, ability of humans. Mirror neurons might explain why we yawn when we see others yawning, and how it's possible that a 4 day old baby (who has never seen her own face), can poke her tongue out in imitation of an adult making that gesture? And they would account for the fact that merely listening to speech modulates the excitability of neurons for muscles of the tongue. More generally, the effect of miming the lip and tongue movements of others when they speak when combined with mirror-based empathy, strengthens the case for a bio-cultural or Baldwinian account of language's evolution as advocated by Terrence Deacon (Deacon 1997). According to this, biological evolution and cultural change are folded into each other: practices, such as gestural mimesis, exert selectional pressure on neurological development, which in turn facilitates further cultural practices, and so on. This adds weight to recent attempts mentioned earlier (eg Armstrong et al 1995) to show that spoken language – in particular the vexed issue of the origin of syntax – can be derived from a prior gestural matrix.

Evidently, mirror neurons require a re-writing of the relation between motor action and perception: *imagining* an action becomes akin to perceiving it, suggesting the existence of a neuronal link between virtual and real activity. And indeed, soon after the discovery of mirror neurons, neuroscientist J. Decety asked: "Do imagined and executed actions share the same neural substrate?" His answer, based on comparing the two sorts of action with respect to their timing, their autonomic responses, and their cortical blood flow, was a cautious yes.

SLIDE 11 fMRI scans of primary motor cortex during actual and imagined gesture with the right hand.

Subsequently, Decety and his colleagues extended this sharing of neural substrate to a more comprehensive hypothesis asserting wshat they called the 'functional equivalence' of producing an action, imagining it, verbalizing it, and observing it.

If we accept that mathematics is fundamentally a form of thought experiment (as I've argued elsewhere, Rotman 2000), a principal element of which is to imagine actions (gestures, journeys, processes), then the two forms of equivalence – motor and functional – work to dissolve any absolute gap between mathematical abstractions (in the head) and their formulation (on the page, in the air). Instead there is a back and forth movement between real actions (observed or executed) and imagined ones in which the actions, by virtue of motor equivalence, can be expressed by "different effector systems". Taken together, the two principles of equivalence offer a theoretical basis for what one might call a motor theory of mathematics, which would integrate into a single ecology the mix of talking, imagining, gesturing, symbolizing, drawing, and eye-scanning found to be present within the learning of mathematics.

Thus gesture, talk, and diagrams, far from being mere epiphenomenal aids to learning and understanding mathematical ideas (the orthodox viewpoint) – would instead be intrinsic to the mathematical content of ideas, to their meaning; indeed, in some cases, would <u>be</u> their meaning in their own – verbal, gestural, visual -- modalities.

Interestingly, a theory with elements akin to this, proposing a transfer of motor-originated meaning across modalities, has been developed to account for the evolution and functioning of human languages

SLIDE 12 *The Motor Theory of Language Origins*
"In the evolution of language, shapes or objects seen, sounds heard, and actions perceived or performed, generated neural programs which, on transfer to the vocal apparatus, produced words structurally correlated with the perceived shapes, objects, sounds and actions."

"The gesture associated with the meaning of any word can be observed by <u>mentally</u> transferring the sound-structure of the word (the articulatory gesture) to the musculature of the arms."

"The forms of individual words are not arbitrary but directly derived from and related to the meaning of the words."
According to his theory, the motor program which generates a word -- an articulatory gesture-- also generates an equivalent body gesture, leading to the claim that for "a different word in a different language for the same meaning, a <u>similar final gesture</u> is generated by a different intermediary gesture associated with different speech-sound elements going to form different words."

All told, Allott presents a heterodox linguistic theory which makes striking and radical claims. To assert that the sound-form and semiotic content of a word are linked is to run foul of a basic tenet -- dogma -- of mainstream 20th century linguistics, which from de Saussure onwards has maintained that arbitraryness rules: that, apart from onamatopoeia-like effects, no systematic connection between phonology and semantics, between how a word sounds and what it means, exists in human languages. Likewise, to assert, as he does, that one can 'limb-speak' a word – can transfer its associated gesture from the articulatory apparatus to one's arms and hands -- is to make the principle of motor equivalence work in an unexpected direction that adds an intriguing layer to Bulwer's extraordinary list of manual gestures.

I've introduced Allott's ideas for language because they run parallel in some respects to the motor theory of mathematics suggested here. I should add, however, that his theory rests on another, well-attested motor theory – that of speech – according to which the <u>perception</u> of speech (and not just its production) consists of processing it as a series of articulatory gestures. Matters are more indirect in the case of mathematics, in that its two-dimensional syntax exceeds that governing speech. Further comparison would take us outside the scope of my task here. Instead I'll simply display some of the diagrams Allott uses to expound his theory.

SLIDE 13 Motor/Speech-Sound groups

To still any lingering doubts that motor actvity and mathematical ideas are intimately linked, let me very briefly refer to a cognitive approach.

SLIDE 14 Where Nathematics Comes From: How the Embodied Mind Brings Mathematics Into Being. George Lakoff & Rafael Nunez (2000)
"All mathematical content resides in embodied mathematical ideas"
"A large number of the most basic, as well as the most sophisticated, mathematical ideas are metaphorical in nature." (364)
"The same neural structure used in the control of complex motor schemas can also be used to reason about events and actions." (35)

The contention here is that mathematics is based on a network of inferences derived from metaphors or schemas of body activity. For example, schemas of starting or stopping or continuing a motion, of moving along a path, etc., are seen as the constituents of elementary arithmetical reasoning; a schema of 'containing' things is seen as the basic metaphor behind the universal language of sets; and so on. There are also "linking metaphors" which combine these schemas into more complex mathematical structures. Overall, the approach reveals links between apparently disparate mathematical ideas and valuably foregrounds embodied motion as an essential strand in the genesis of mathematical ideas. However, it does so via an analysis which objectifies thinking/imagining processes –- unfolding events in Whitehead's sense -- into static structures. Thus, for example, the gesture of cupping the hands is abstracted into an object of cognitive science (the container schema), to become an after-the-fact entity detached from the original gesture's motoric affiliations with other gestures (gathering, holding, separating, carrying, etc), and thereby rendered opaque to any understanding of its possible realization in other modalities. It also misses the important interaction that exists between gestures and mathematical diagrams.

Let me enlarge this last point in a theoretical direction by citing the work of Gilles Chatelet and Maurice Merleau-Ponti.

First, the embodied phenomenology of Merleau-Ponti (as distinct I should say from the disembodied – transcendental – phenomenology of Edmund Husserl). A triangle, for example, is normally defined as a 3-sided figure. But, Merleau-Ponti insists, "There is no definition of a triangle which includes in advance the properties subsequently to be demonstrated", no "logical definition of the triangle could equal in fecundity the vision of the figure." (441) On the contrary, the creative force, their ability to mediate new meanings, of mathematical entities such as triangles is pre-formal, inseparable from our lived, embodied interaction with them. A triangle's essence is physical, concrete, a "certain modality of my hold on the world." (442) And this literally so: a hold consisting of the drawing and perceiving gestures which determine any diagram, but which is never exhausted by them, since, as Merleau-Ponti points out, the gestures overflow any particular diagrammatic representation. Thus, after presenting the most basic geometrical construction about the angles of a triangle, he observes

"My perception of the triangle was not ... fixed and dead ... it was traversed by lines of force, and everywhere in it new directions, not traced out yet, came to light. In so far as the triangle was implicated in my hold on the world, it was bursting with indefinite possibilities of which the construction actually drawn was merely one." (443)

Merleau-Ponti's triangles are flat, zero curvature Euclidean ones, their angles sum to 180 degrees. There are also, as we know, non-Euclidean geometries: spherical surfaces with positive curvature, where the sum is greater than 180, and hyperbolic surfaces with negative curvature, where the angles of any triangle always sum to less than 180 degrees. His remarks, however, apply to all triangles.

SLIDE 15 2- and 3-dimensional diagrams of hyperbolic space.

Finally, let me turn to the work of Gilles Chatelet on the mathematisation of space, for whom the fecundity of diagrams, their ability to mobilize Merleau-Ponti's "indefinite possibilities" of mathematical meaning, derives from their relation to gestures.

SLIDE 16
"A diagram can transfix a gesture, bring it to rest, long before it curls up into a sign."
"Gesture refers to a disciplined distribution of mobility before any transfer takes place: one is infused with the gesture before knowing it."
"Gesture is not substantial: it gains amplitude by determining itself." (9-10)

Two principles organize Chatelet's genealogy of physico-mathematical space. One is the insistence on intuition and premonition, on the 'metaphysical' or contemplative dimension of mathematical thought; the other the insistence that mathematical abstraction cannot be divorced from "sensible matter", from the movement and agency of bodies.

In Chatelet's account, mathematics arises in the traffic between embodied rumination, "figures tracing contemplation", and defined abstractions "formulae actualizing operations" (7). The vehicles that articulate the two realms, that carry this traffic, are metaphors. They function to form bridges "from premonition to certainty" (and, in doing so accomplish a "shedding [of] their skin" whereby they metamorphose into operations. (9)

Diagrams are quite different. They are figures of contemplation and rumination and can, like mandalas, focus attention, heighten awareness, and literally embody thought. Here is a selection:

SLIDES 17 – 20, 21 mathematical diagrams 1-4, and so on.

Diagrams, for Chatelet, occur in relation to gestures. They are immobilized gestures that "distill action". Unlike metaphors, whose action exhausts them, diagrams do not disappear in being used. "When a diagram immobilizes a gesture in order to set down an operation, it does so by sketching a gesture that then cuts out another."(10) This capacity of diagrams makes them sites for the relay and retrieval of gestures and, as such, amenable to thought experiments – imagined narratives which allow new, mathematical spaces and operations into being whose stability is guaranteed by the repeatability of their underlying gestures.

For Chatelet, gesture operates across the board. Globally: gestures periodize the history of spatialization, "Gesture and problems mark an epoch" (3). Thus, the solenoid, for example, "unwinding a loop around a channel" created electro-dynamic space by solving the metaphysical problem presented by the co-presence of magnetism and electricity.(155) And locally: as a constituent of mathematical thought: "The concept of gesture seems to us crucial in our approach to the amplifying abstraction of mathematics." (9); a mode of abstraction that cannot be captured by formal systems, which, he insists, "would like to buckle shut a grammar of gestures." One is infused with a gesture "before knowing it": its action is a-signifying, it eludes the mode of operation of signs: since a gesture is not referential, "it doesn't throw out bridges between us and things", it is not predetermined, "no algorithm controls its staging", and it is "not substantial" in that it generates meaning through the fact and the manner of its taking place. It is performative, enactive.

But enough. Chatelet is difficult, his ideas subtle, and my time is up. I hope the narrative gallop here from chirology and gesticulation to immobilized gestures in diagrams, through the ecology of mathematics learning, eye movements, mirror neurons, the motor theory of language, embodied minds, and phenomenological triangles, illustrates the existence of deep-lying links as well as the possibility of a fruitful interplay between mathematical ideas and disciplined mobilities of the body.

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