Fibonacci… and his rabbits

Leonardo Pisano Fibonacci is best remembered for his problem about rabbits. The answer – the Fibonacci sequence -- appears naturally throughout nature.

But his most important contribution to maths was to bring to Europe the number system we still use today.

In 1202 he published his *Liber Abaci* which introduced Europeans to the numbers first developed in India by the Hindus and then used by the Arabic mathematicians… the decimal numbers.

We still use them today.

OK, OK… Let's talk rabbits…

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Suppose that our rabbits never die. And the female always produces one new pair (one male, one female) every month from the second month on.

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The puzzle that I posed was...

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Suppose that our rabbits never die. And the female always produces one new pair (one male, one female) every month from the second month on.

The puzzle that I posed was...

How many pairs will there be in one year?

Pairs

1 pair

At the end of the first month there is still only one pair

Pairs

1 pair

End first month… only one pair

1 pair

At the end of the second month the female produces a new pair, so now there are 2 pairs of rabbits

2 pairs

Pairs

1 pair

End first month… only one pair

1 pair

End second month… 2 pairs of rabbits

2 pairs

At the end of the third month, the original female produces a second pair, making 3 pairs in all in the field.

3 pairs

Pairs

1 pair

End first month… only one pair

1 pair

End second month… 2 pairs of rabbits

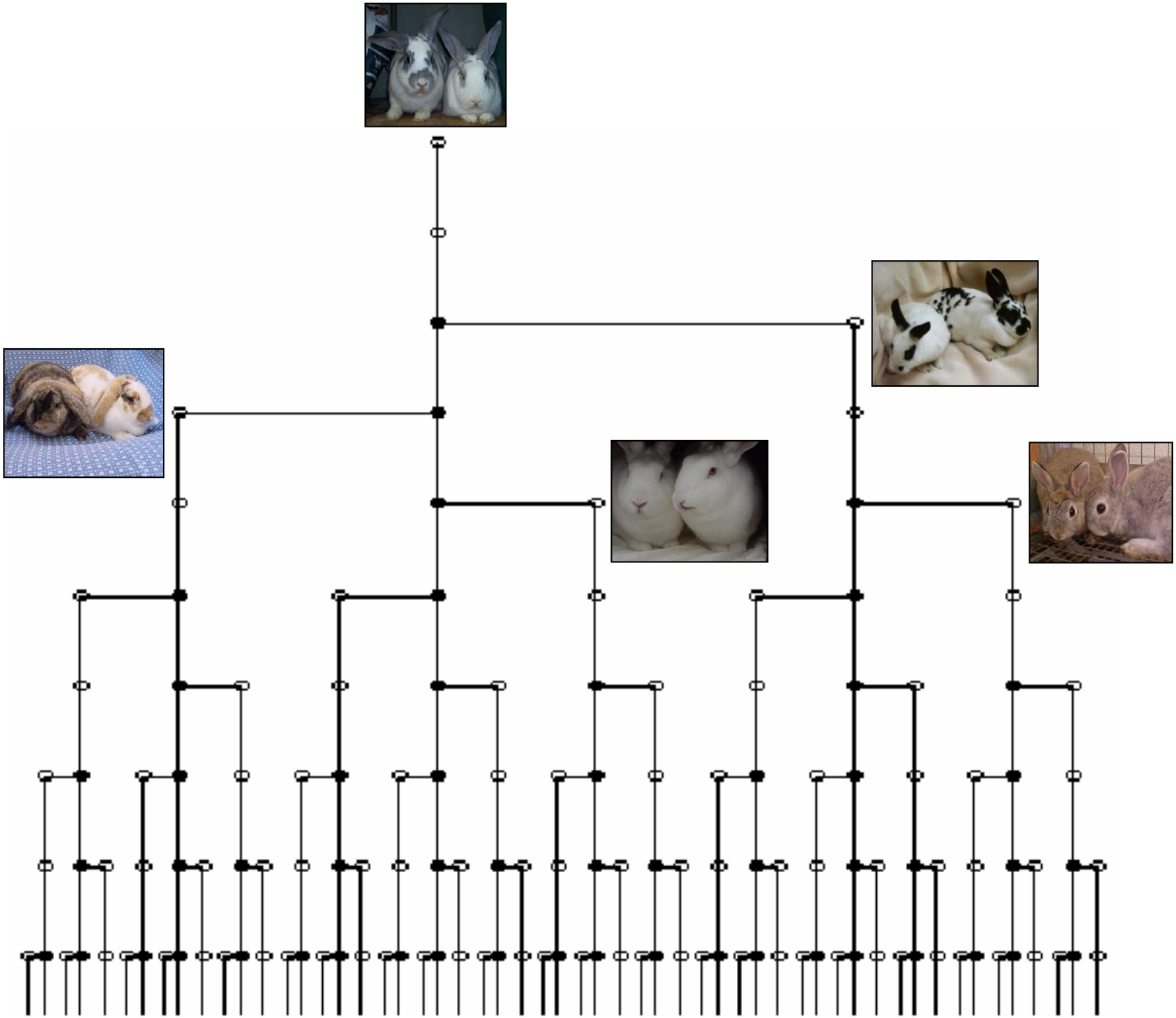
2 pairs

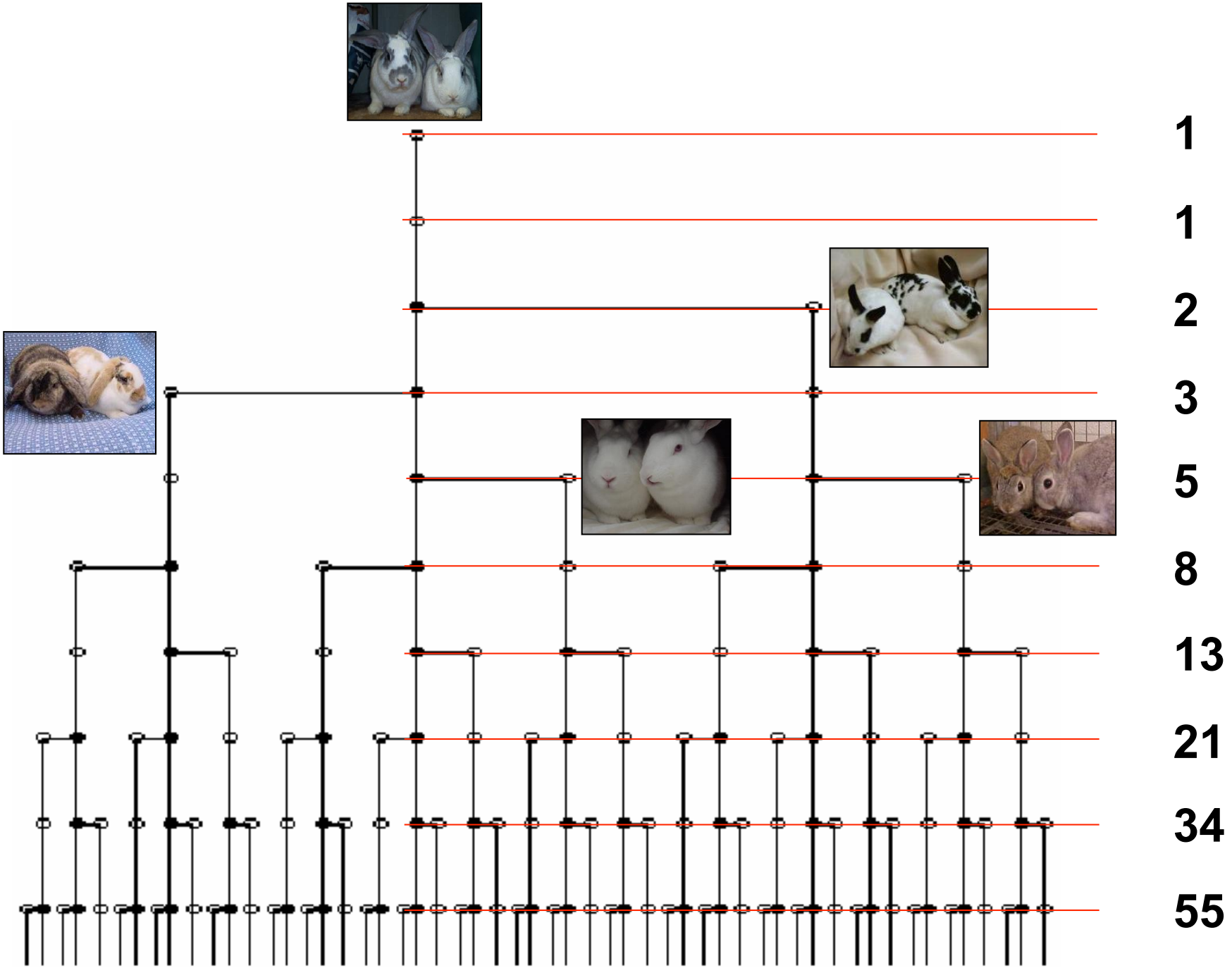
End third month… 3 pairs

3 pairs

5 pairs

At the end of the fourth month, the first pair produces yet another new pair, and the female born two months ago produces her first pair of rabbits also, making 5 pairs.




1
1
2
3
5
8
13
21
34
55

1	1
2	1
3	2
4	3
5	5
6	
7	
8	
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Dudeney… and his cows

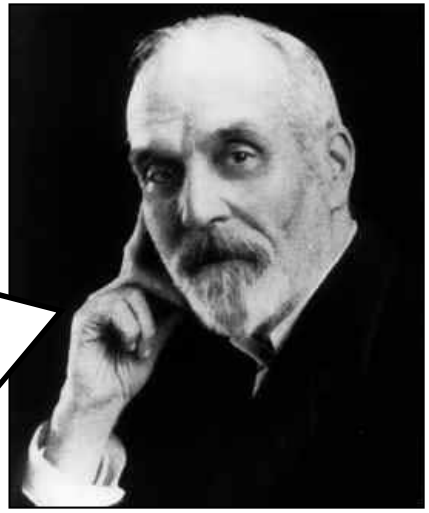
Henry Dudeney sent his life thinking up maths puzzles.

Instead of rabbits, he used cows.

He notices that really, it is only the females that are interesting - er - I mean the *number* of females! He changes months into years and rabbits into bulls (male).

If a cow produces its first she-calf at age two years and after that produces another single she-calf every year, how many she-calves are there after 12 years, assuming none die?

'The history of mathematical puzzles entails nothing short of the actual story of the beginnings and development of exact thinking in man. Our lives are largely spent in solving puzzles; for what is a puzzle but a perplexing question? And from our childhood upwards we are perpetually asking questions or trying to answer them.'



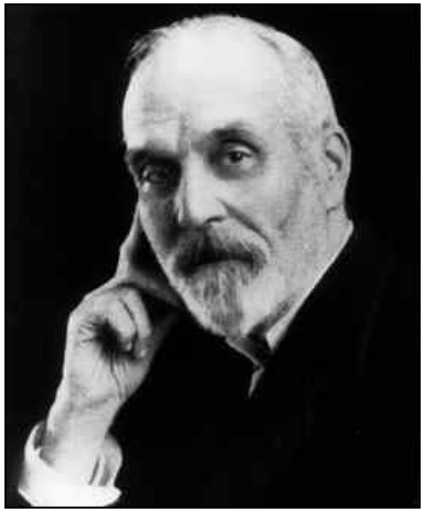
Three countrymen met at a market. "Look here, " said Hodge to Jakes, "I'll give you six of my pigs for one of your horses, and then you'll have twice as many animals here as I've got."

"If that's your way of doing business," said Durrant to Hodge, "I'll give you fourteen of my sheep for a horse, and then you'll have three times as many animals as I."

"Well, I'll go better than that," said Jakes to Durrant; "I'll give you four cows for a horse, and then you'll have six times as many animals as I've got here."

How many animals did the three take to market?

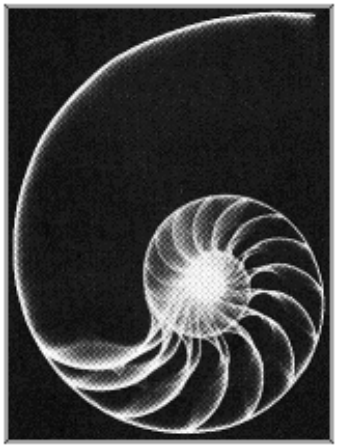
Dudeney… and his cows

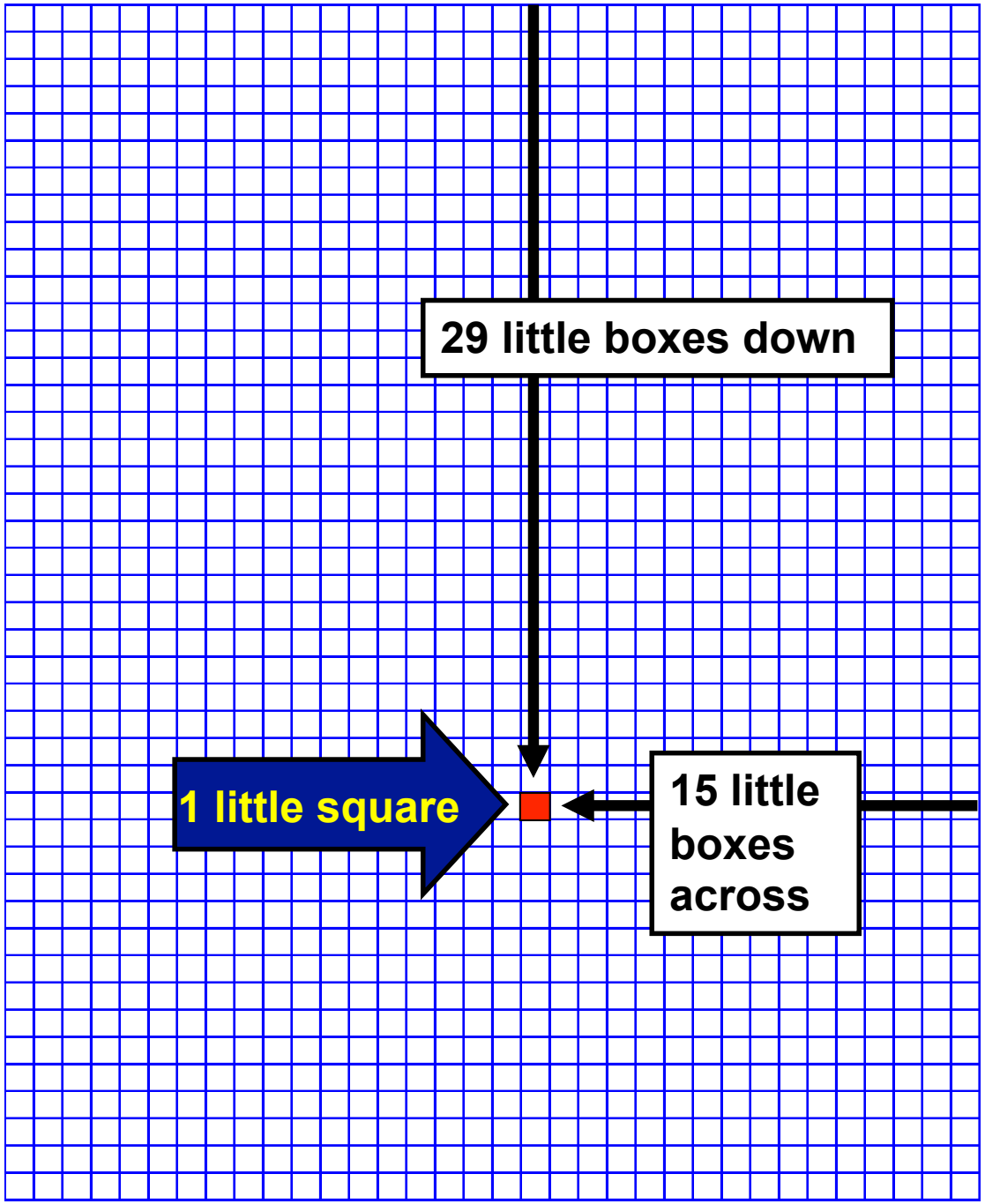


If a cow produces its first she-calf at age two years and after that produces another single she-calf every year, how many she-calves are there after 12 years, assuming none die?

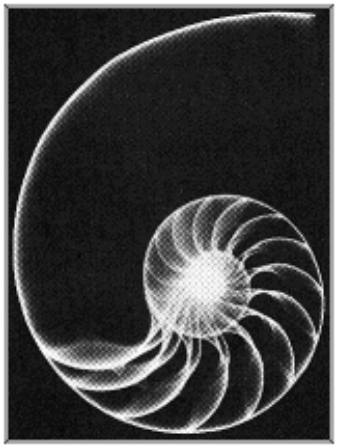
End year 1	0 she calves
2	1 she calf
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

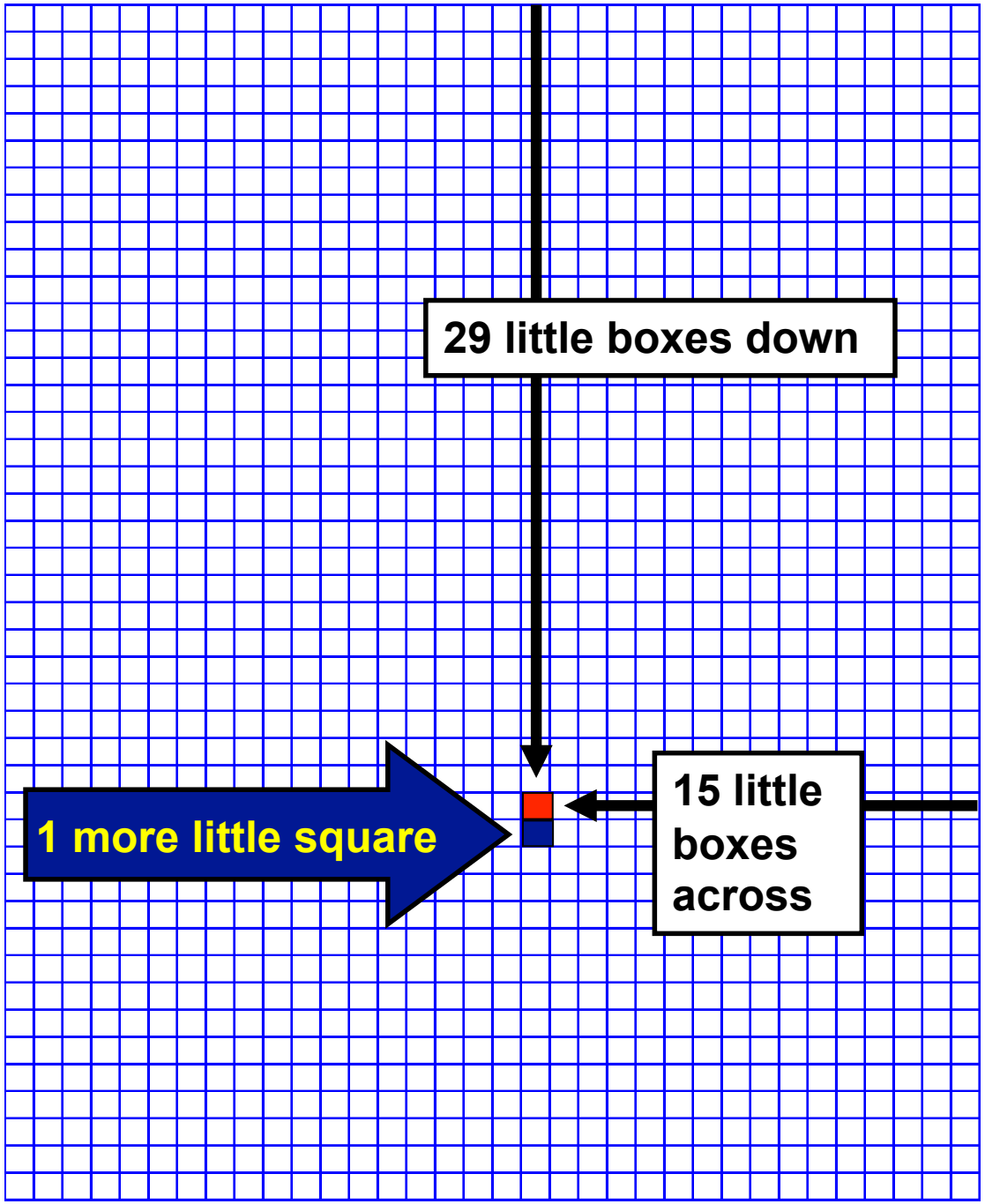
It used to be told at St Edmondsbury that many years ago they were so overrun with mice that the good abbot gave orders that all the cats from the country round should be obtained to exterminate the vermin. A record was kept, and at the end of the year it was found that every cat had killed an equal number of mice, and the total was exactly 1111111 mice. How many cats do you suppose there were?



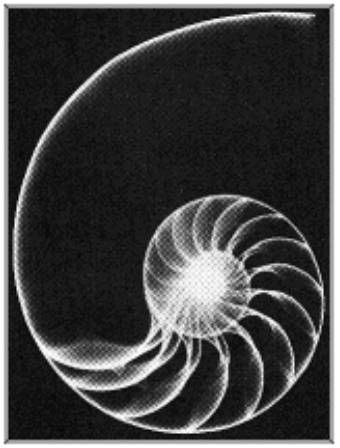


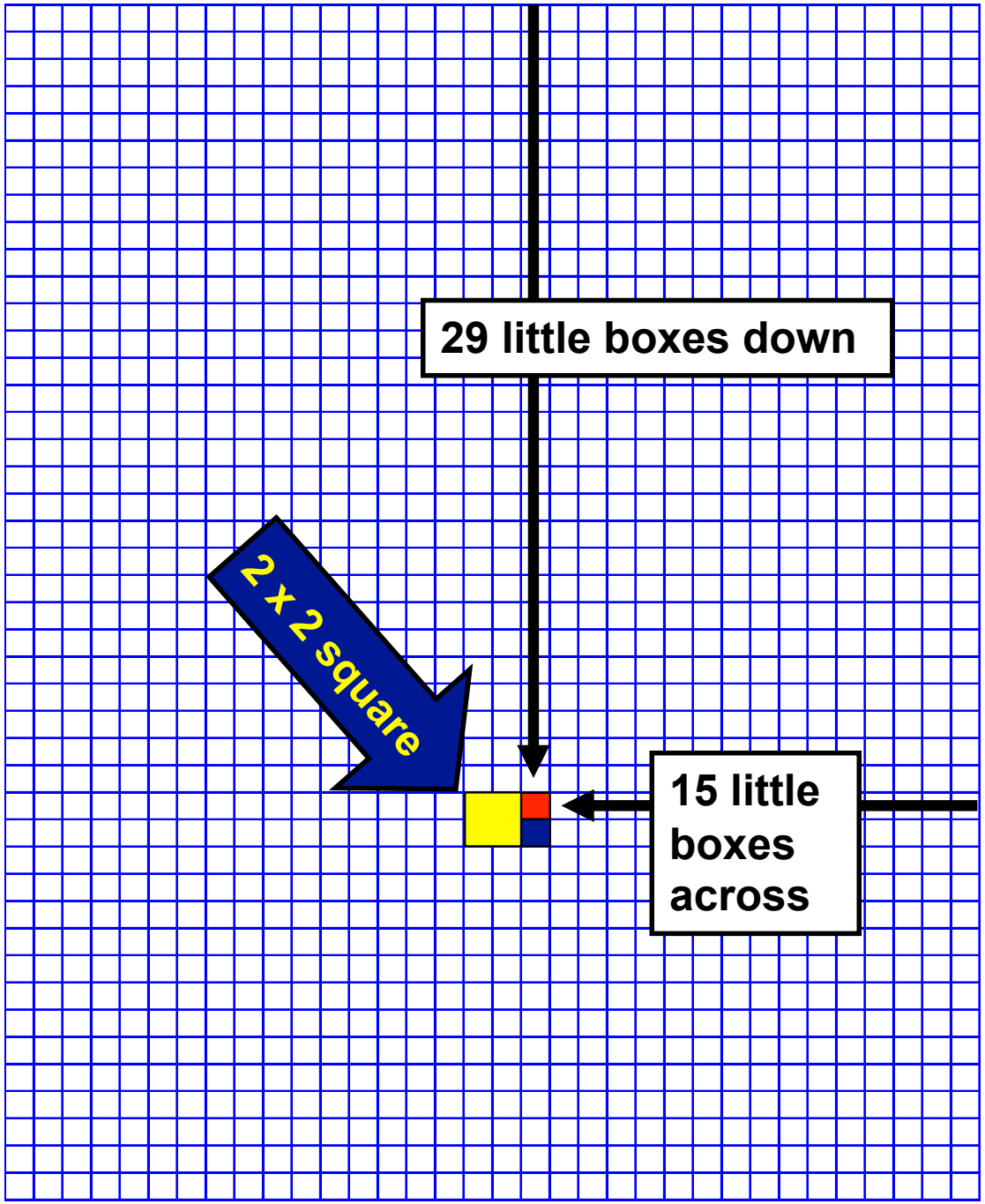
Box	Side
1	1



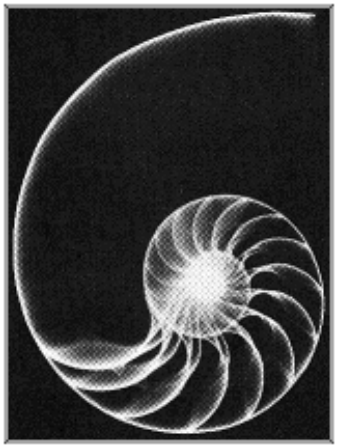


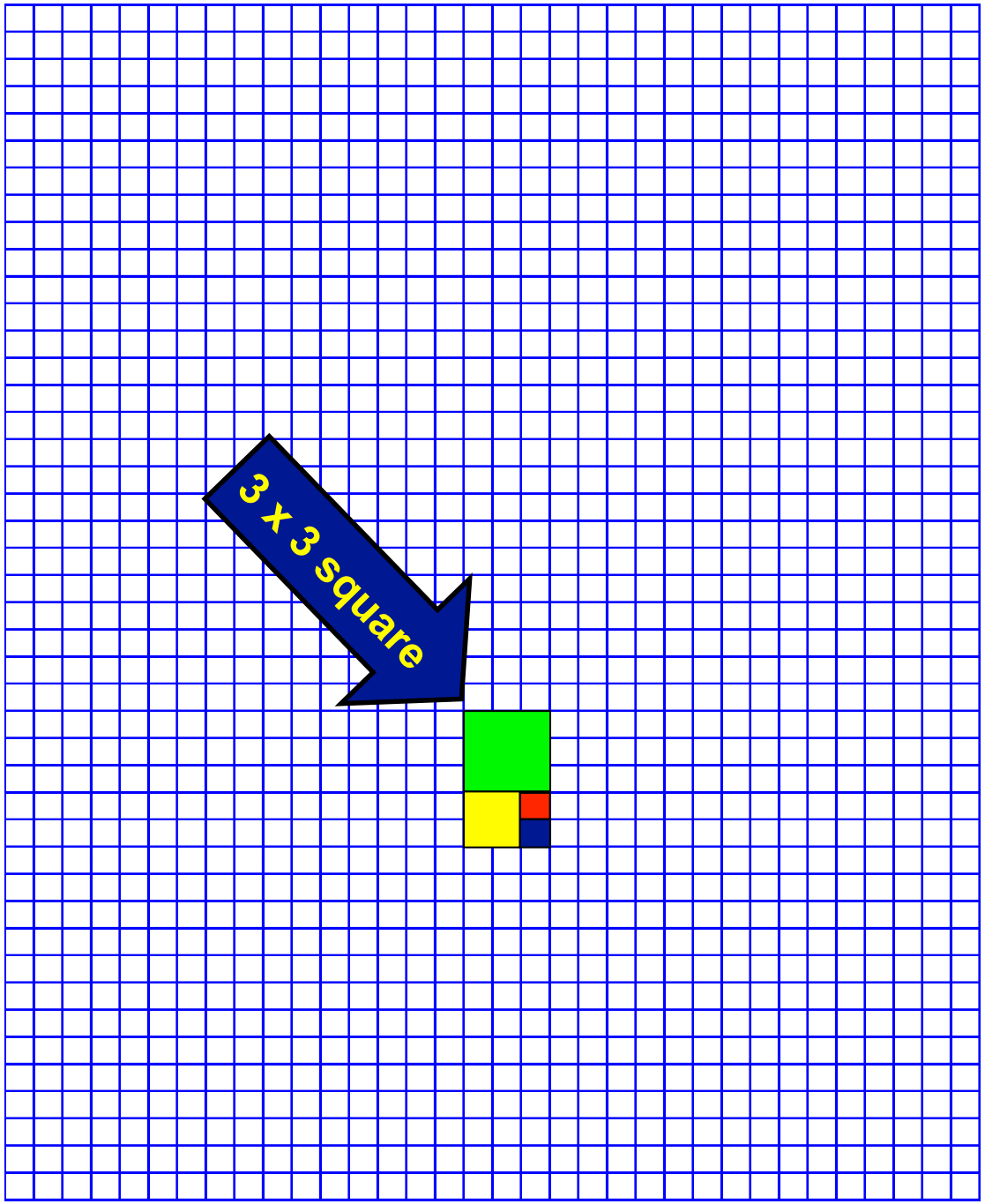
Box	Side
1	1
2	1



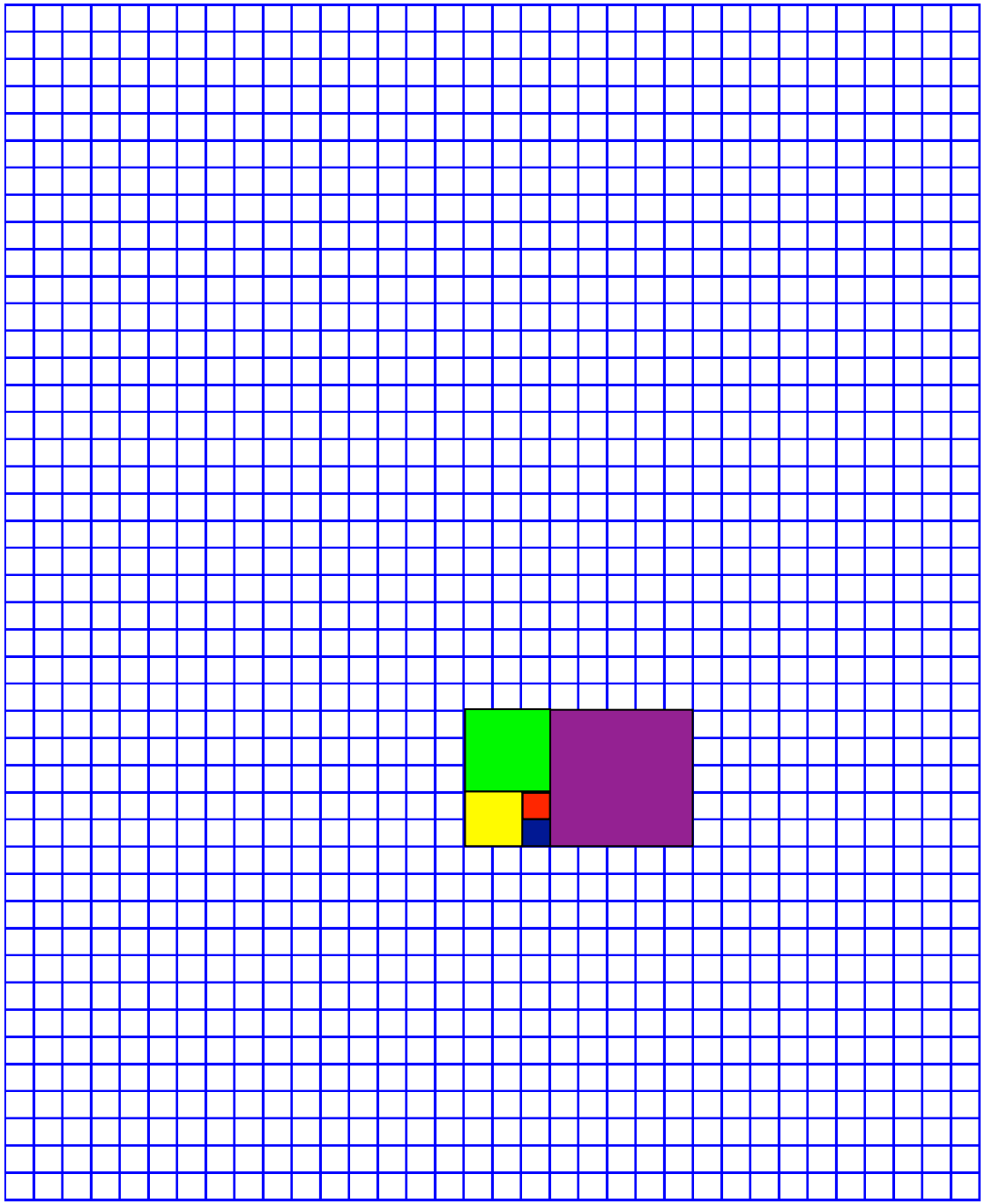


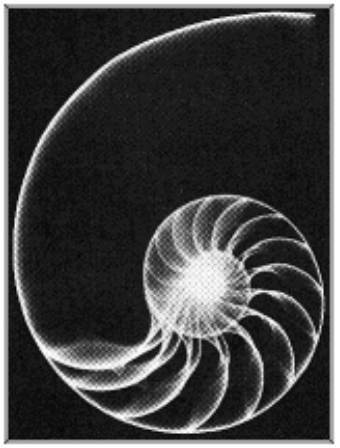
Box	Side
1	1
2	1
3	2



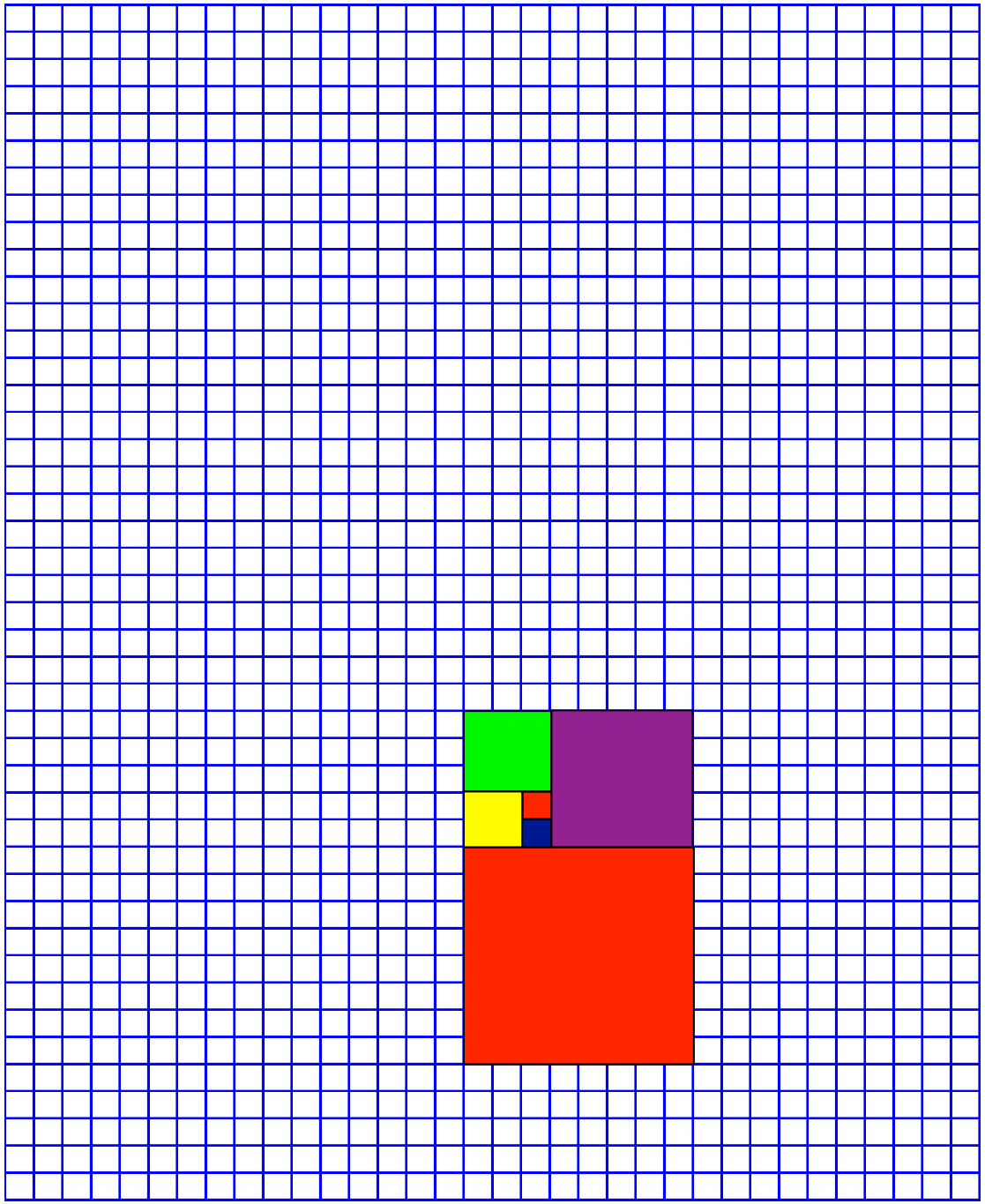


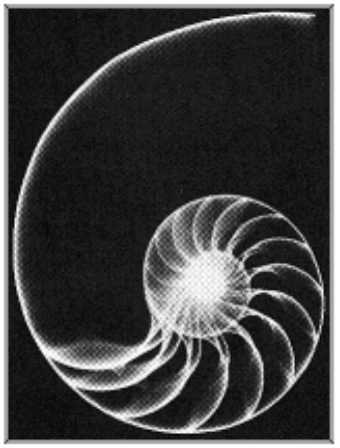
Box	Side
1	1
2	1
3	2
4	



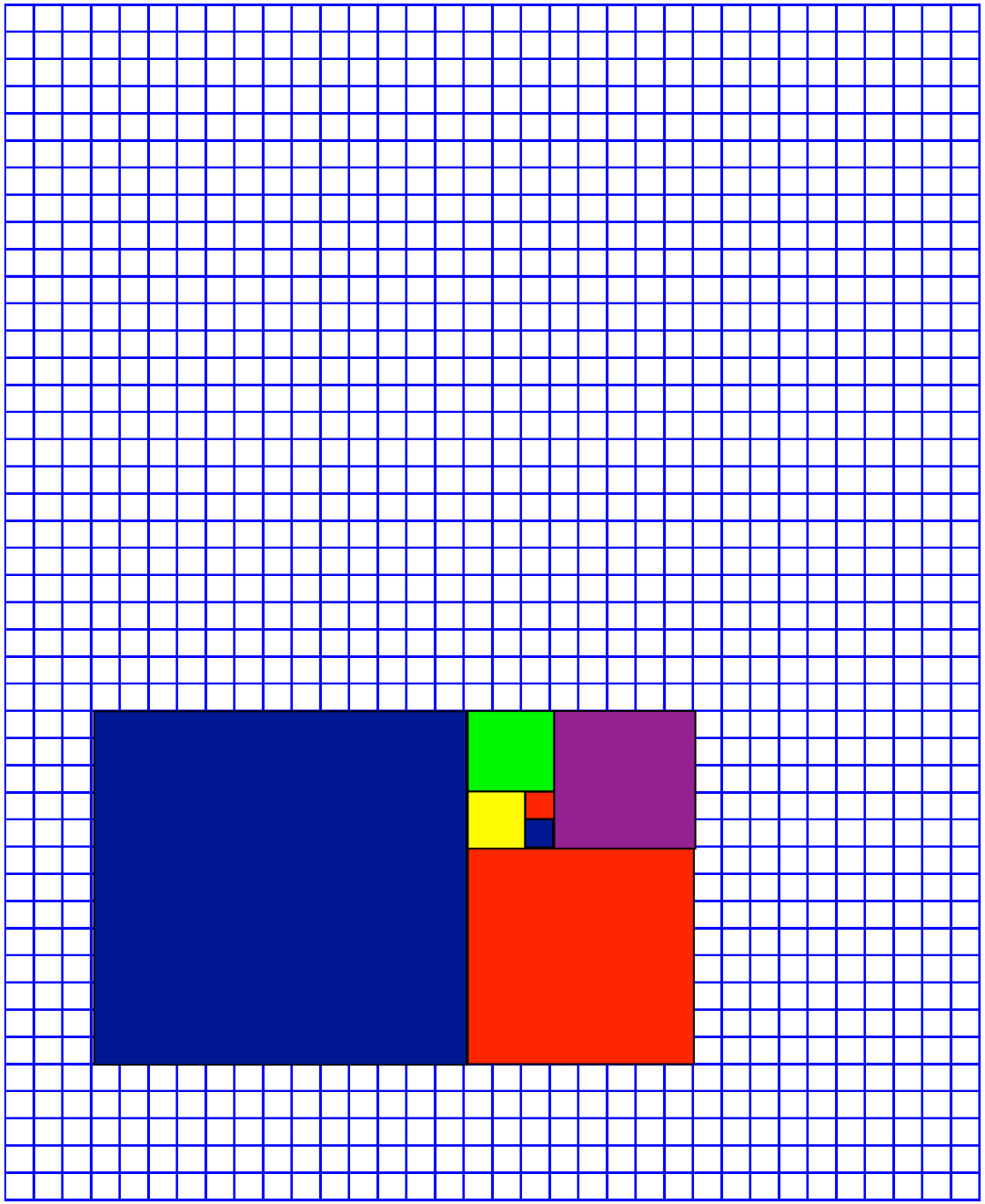


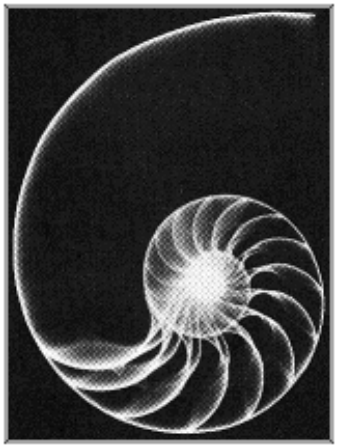
Box	Side
1	1
2	1
3	2
4	



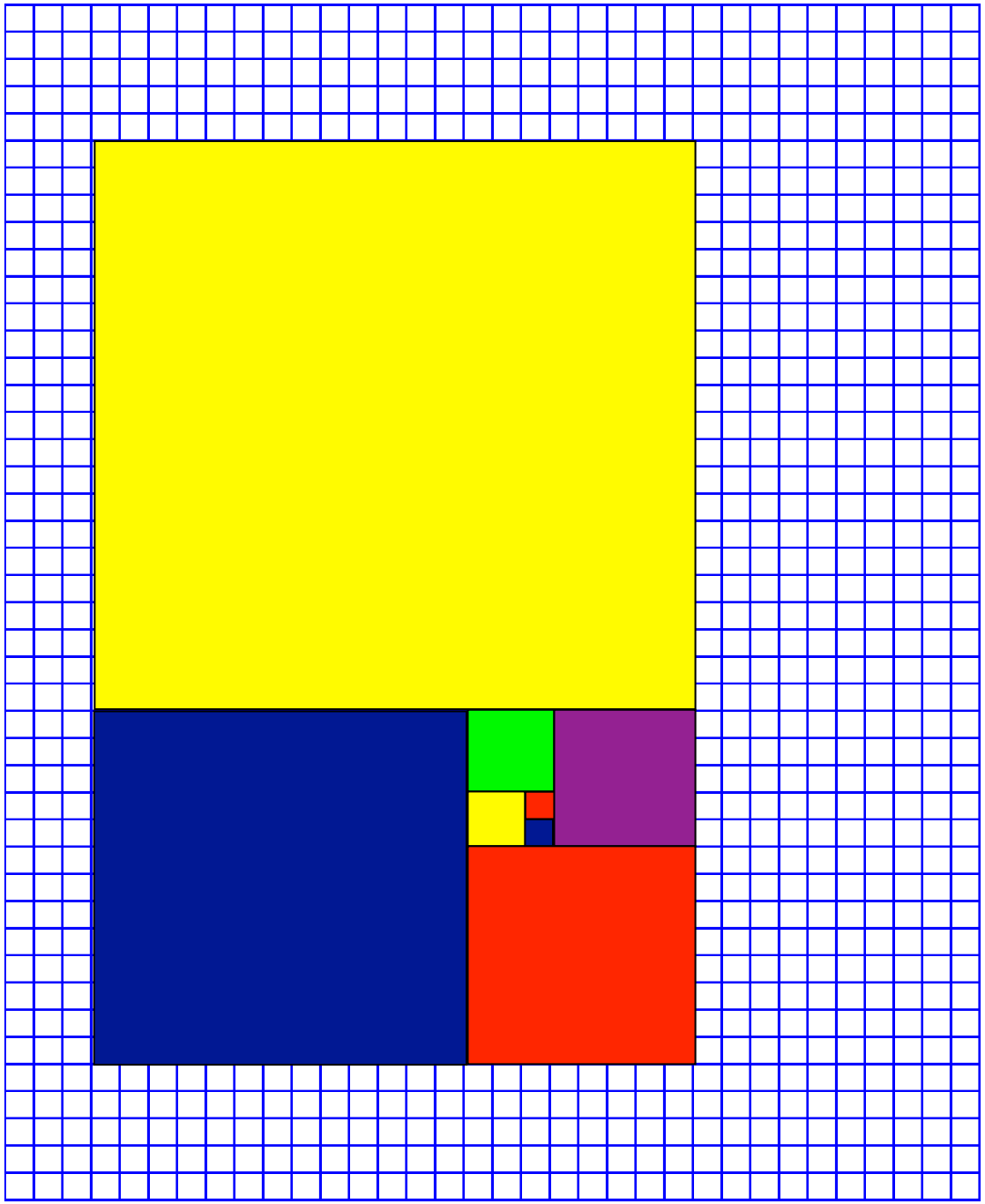


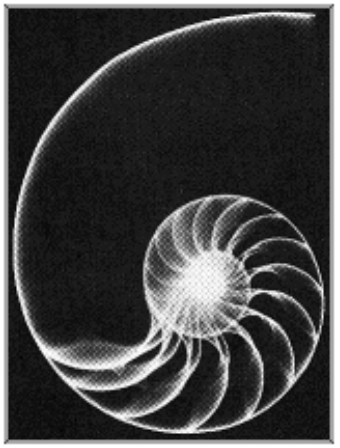
Box	Side
1	1
2	1
3	2
4	



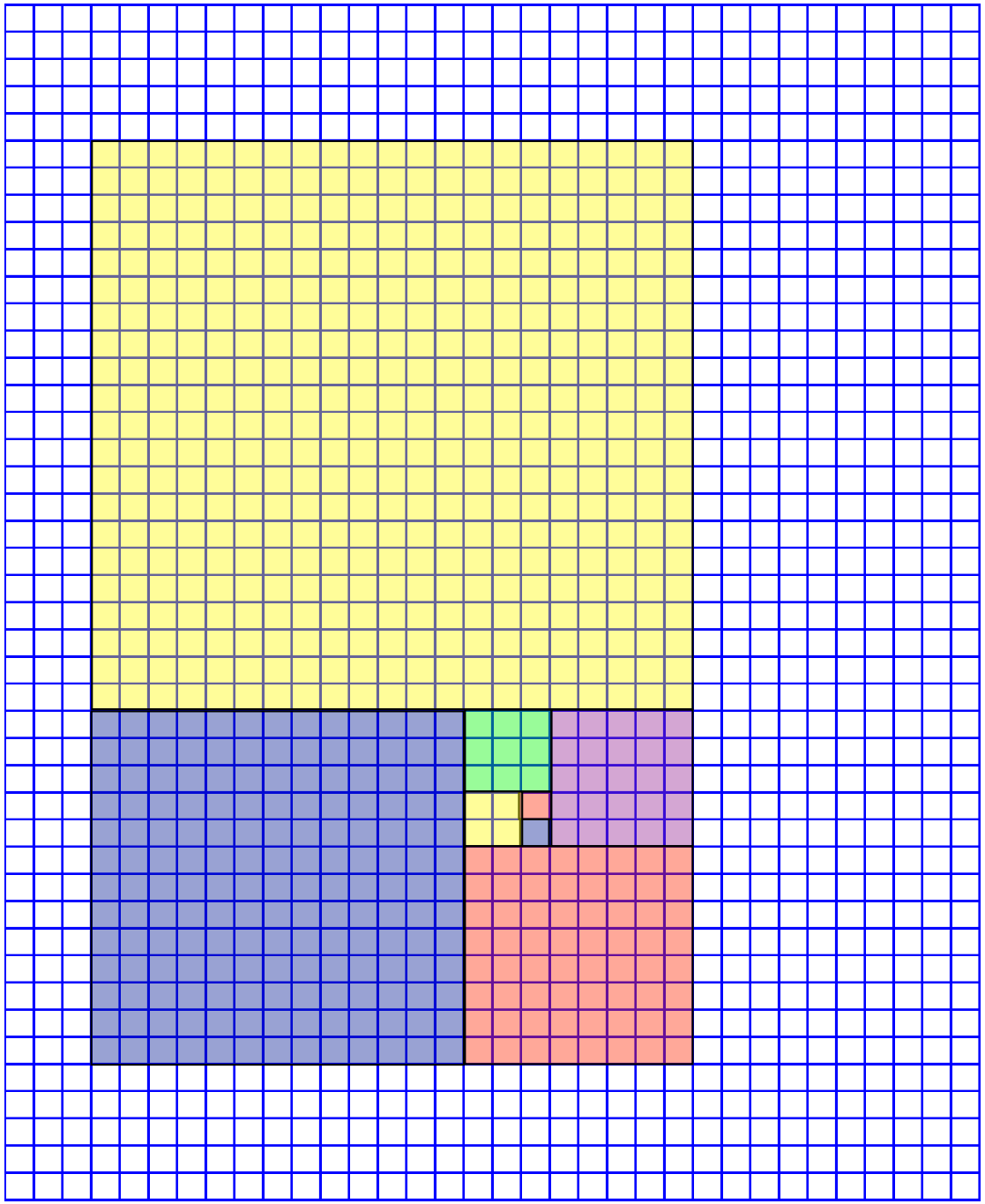


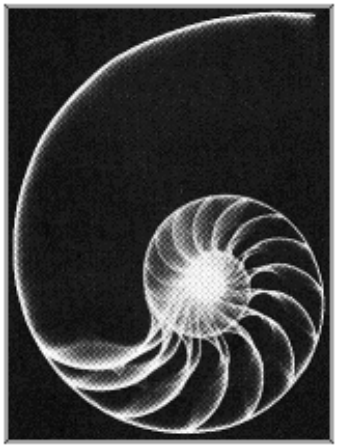
Box	Side
1	1
2	1
3	2
4	





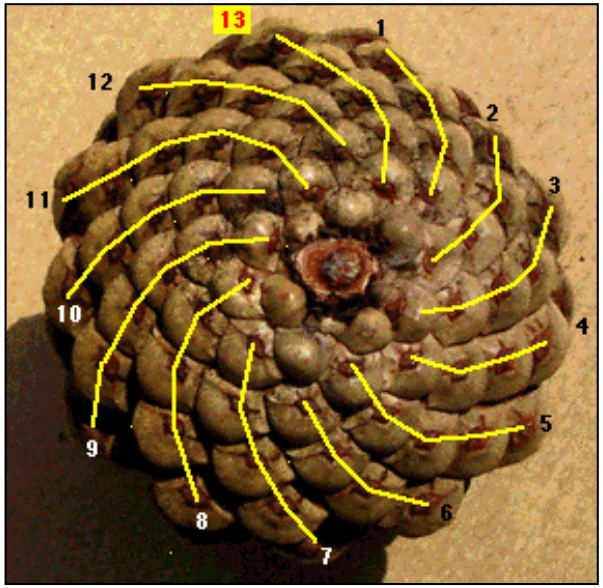
Box	Side
1	1
2	1
3	2
4	

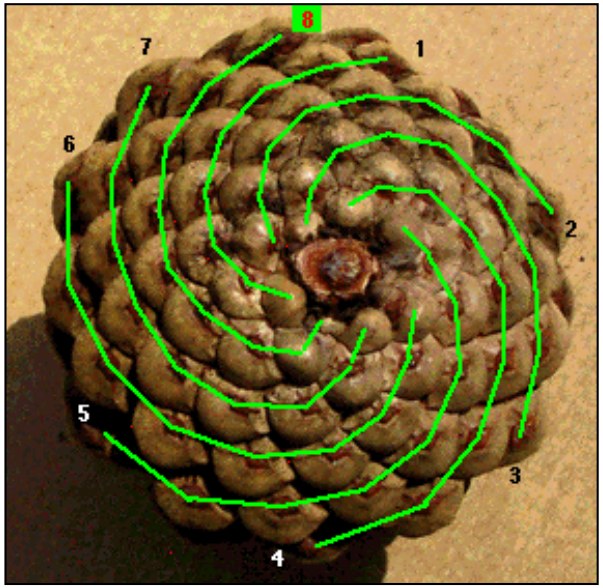


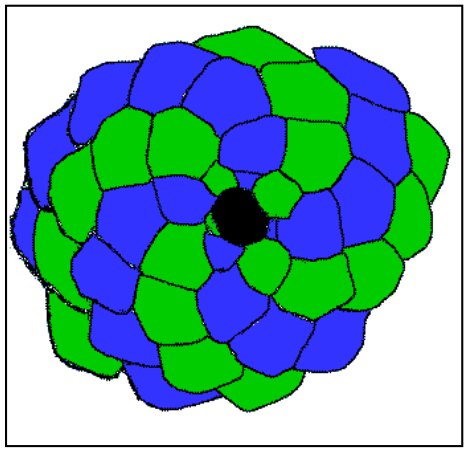


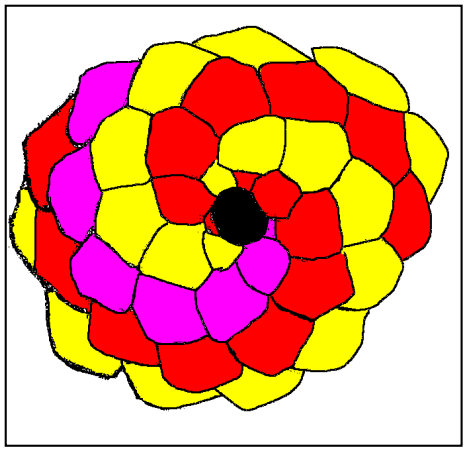
Box	Side
1	1
2	1
3	2
4	

Fibonacci's sequence… in nature









1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584…

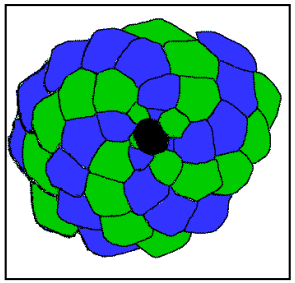
Fibonacci's sequence… in nature

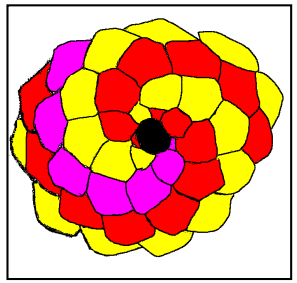
Collect some pine cones for yourself and count the spirals in both directions.

A tip: Soak the cones in water so that they close up to make counting the spirals easier.

Are all the cones identical in that the steep spiral (the one with most spiral arms) goes in the same direction?

What about a pineapple? Can you spot the same spiral pattern? How many spirals are there in each direction?





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Fibonacci's sequence… in nature

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584…

Fibonacci's sequence… in nature

Take a look at a cauliflower next time you're preparing one: Count the number of florets in the spirals on your cauliflower. The number in one direction and in the other will be Fibonacci numbers, as we've seen here. Do you get the same numbers as in the pictures?

Take a closer look at a single floret (break one off near the base of your cauliflower). It is a mini cauliflower with its own little florets all arranged in spirals around a centre.

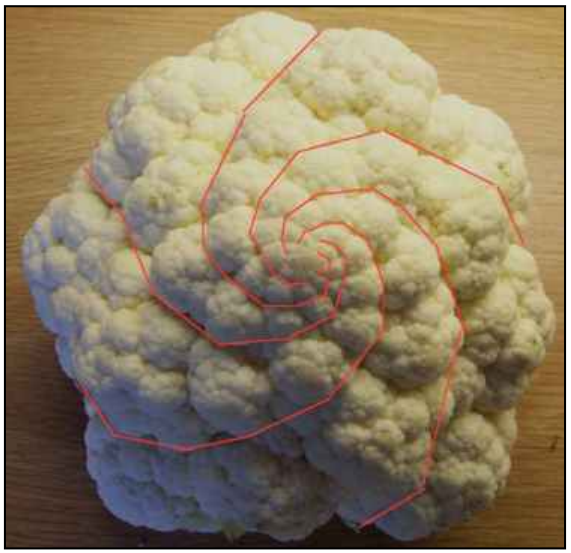
If you can, count the spirals in both directions. How many are there?

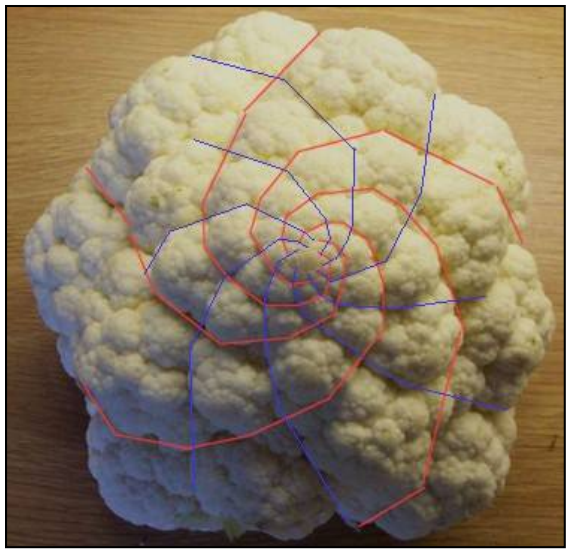
Then, when cutting off the florets, try this: start at the bottom and take off the largest floret, cutting it off parallel to the main "stem".

Find the next on up the stem. It'll be about 0·618 of a turn round (in one direction). Cut it off in the same way.

Repeat, as far as you like and..

Now look at the stem. Where the florets are rather like a pinecone or pineapple. The florets were arranged in spirals up the stem. Counting them again shows the Fibonacci numbers. Try the same thing for broccoli.





1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584…

Fibonacci’s sequence… in nature

Look for the Fibonacci numbers in fruit.
What about a **banana**? Count how many "flat" surfaces it is made from - is it 3 or perhaps 5? When you've peeled it, cut it in half (as if breaking it in half, not lengthwise) and look again. Surprise! There's a Fibonacci number.
What about an **apple**? Instead of cutting it from the stalk to the opposite end (where the flower was), ie from "North pole" to "South pole", try cutting it along the "Equator". Surprise! there's your Fibonacci number!
Try a Sharon fruit.
Where else can you find the Fibonacci numbers in fruit and vegetables?

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584…

Fibonacci's sequence… in nature

On many plants, the number of petals is a Fibonacci number:

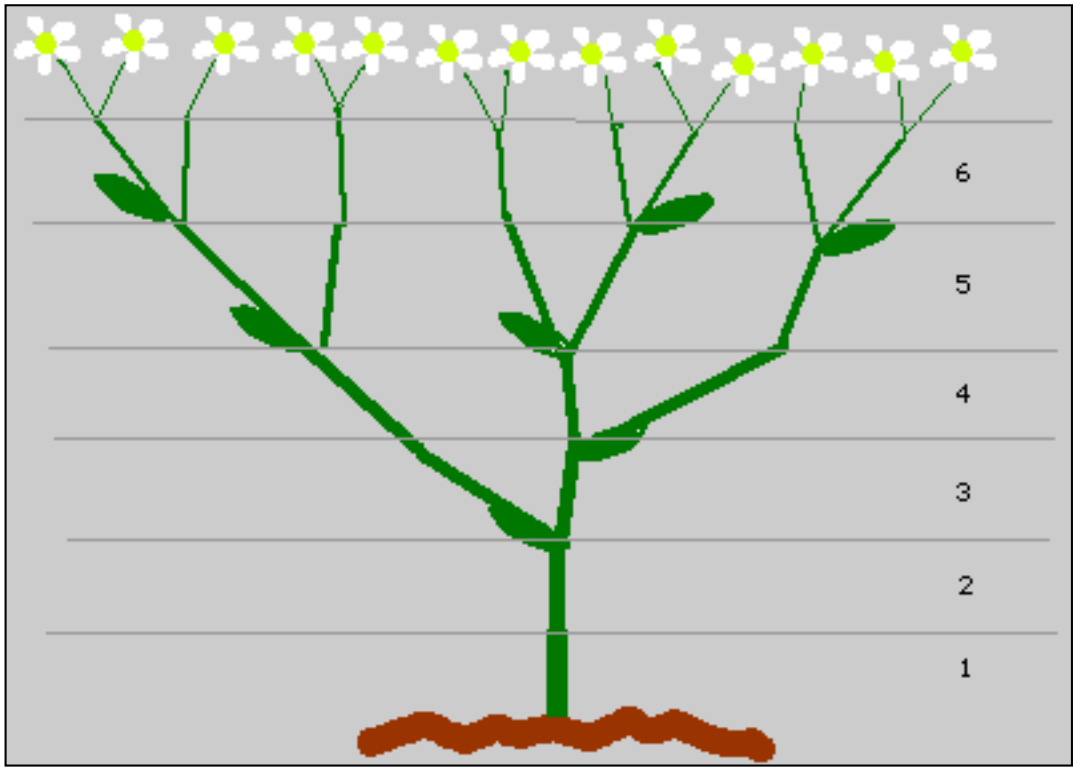
Buttercups have 5 petals; lilies and iris have 3 petals; some delphiniums have 8; corn marigolds have 13 petals; some asters have 21 whereas daisies can be found with 34, 55 or even 89 petals.

13 petals: ragwort, corn marigold, cineraria, some daisies
21 petals: aster, black-eyed susan, chicory
34 petals: plantain, pyrethrum
55, 89 petals: michaelmas daisies, the asteraceae family.

Some species are very precise about the number of petals they have - eg buttercups, but others have petals that are very near those above, with the average being a Fibonacci number.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584…

Fibonacci's sequence… in nature



One plant in particular shows the Fibonacci numbers in the number of "growing points" that it has.

Suppose that when a plant puts out a new shoot, that shoot has to grow two months before it is strong enough to support branching.

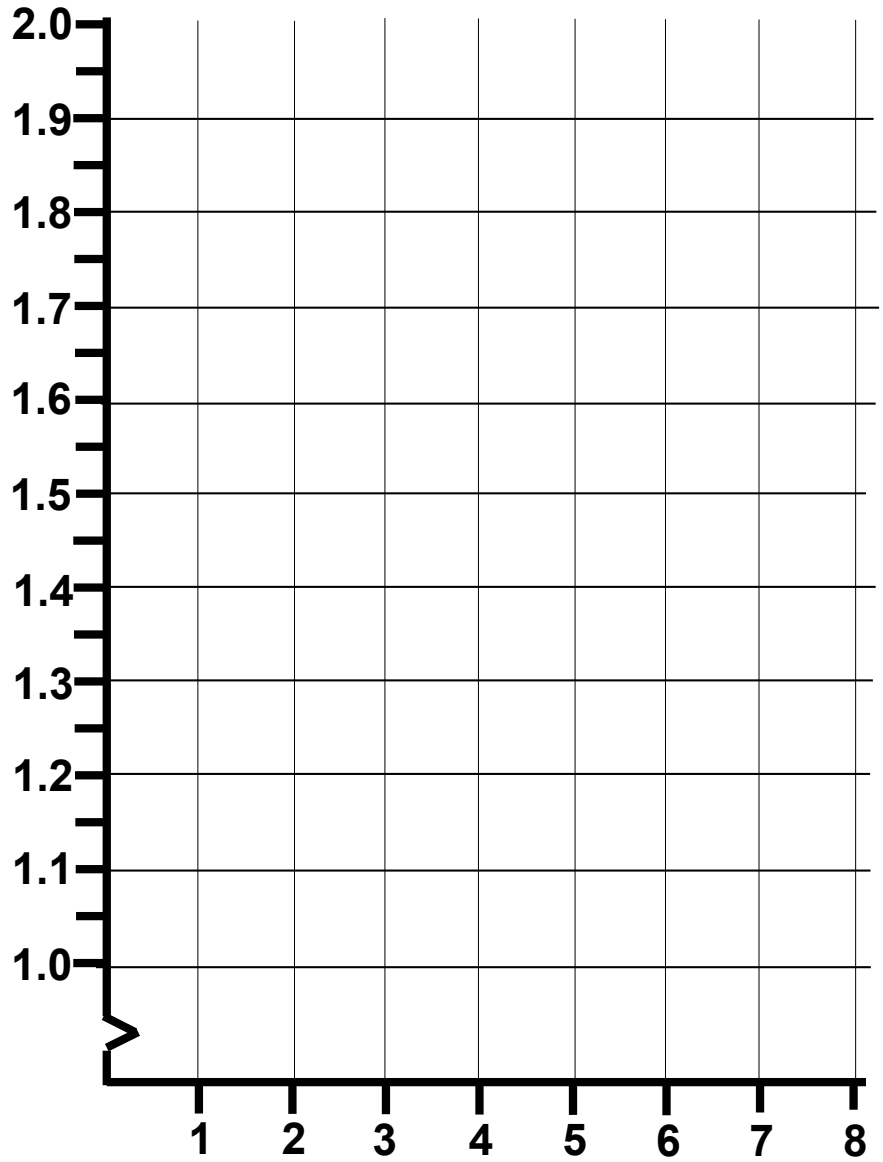
If it branches every month after that at the growing point, we get the picture shown here.

A plant that grows very much like this is the "sneezewort".

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584…

Fibonacci's sequence… in art

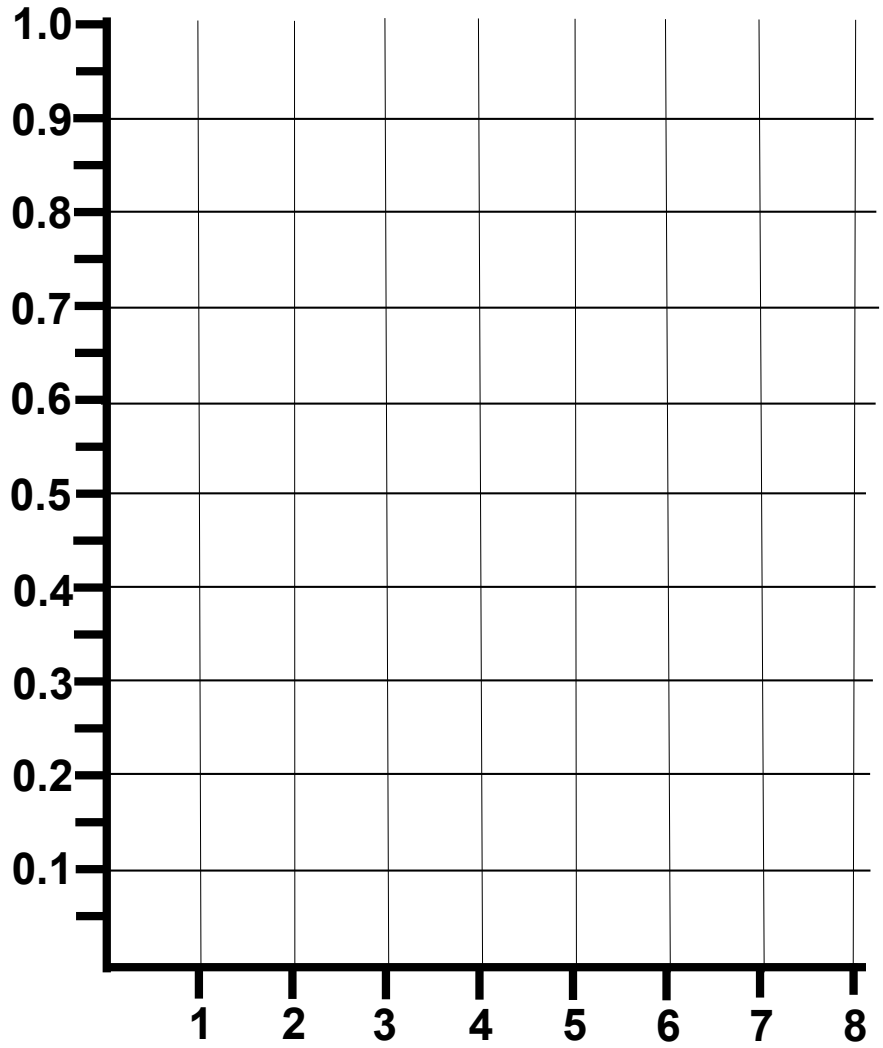
Sequence position n	Fibonacci Number	Fibn/Fib(n-1) = Phi ϕ	Phi ϕ
1	1		
2	1	1/1	1
3	2	2/1	2
4	3	3/2	1.5
5	5	5/3	
6	8		
7			
8			
9			
10			
11			
12			
13			
14			



1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584…

Fibonacci's sequence… in art

Sequence position n	Fibonacci Number	Fibn/Fib(n +1)= Phi ϕ	Phi ϕ
1	1	1/1	1
2	1	1/2	0.5
3	2	2/3	0.6666
4	3	3/5	
5	5	5/8	
6	8		
7			
8			
9			
10			
11			
12			
13			
14			



1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584…


The Golden Ratio
2.5
2
1.5
1
0.5
0
1.618034
0.618034
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
Phi
phi