Mathematics and Biography

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We who love mathematics and who wish to help the general educated public to share our appreciation have a daunting task. So many in our intended audience have painful memories of their encounter with school mathematics. When even the very sight of a formula can generate fear and dismay, what are we to do? Everyone loves stories, and so, we can try to embed our expositions of interesting mathematics in a narrative framework.

Stories can be either true or false: they can be strictly factual, be somewhat exaggerated, or can even be frankly fictional. All of these possibilities have been realized in writing about mathematical subjects. When a fictional tale about mathematical matters commingles truth with falsity, some mathematicians will deplore this, while others will simply delight in the promulgation of a story to the general public that is "good for mathematics". But when an account offers itself as being factual, there is no substitute for accuracy. As the philosopher Richard Kuhns reminds us in a recent monograph:

> Story events are in some ways unlike history events, although in both cases events must follow in a graspable pattern from the opening event. We know the bards of epic tales often changed events as they told stories, without invalidating or making false the stories themselves. In history writing, the historian does not have that latitude. He may exercise it without the constraint of truth if he wishes to do so, but that would make him a bad historian, where it might make the epic storyteller a good bard.
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> When we reflect philosophically on storytelling and its relationship to the writing of history, we question the capacity of historical narration to be in possession of an agreed-upon, shared truth. Such questioning, though, does not take the form of denying the possibility of truth in history but rather suggests that we discover the various ways in which historical narrative can function, and one such way is as a mode of storytelling. There is a close connection between history telling and storytelling. As stories were being brought to birth, mysteriously in so many cases, humankind relied on them to record its history and also to answer its questions …[1]

A key problem is that too much mathematical exposition for the general public is either written by experts who are not wonderful writers or by writers who don't really understand what they are writing about. I limit myself to two particularly egregious examples of the latter difficulty:

In their *Gödel: A Life of Logic*[2], John L. Casti and Werner De Pauli attempt an account of Gödel's life and thought. I quote from my review of their book:

> Because Gödel's work on undecidability is of such general interest, treatments of his life and work intended for a general audience are very desirable. The book by Casti and DePauli being reviewed is an effort in this direction. Unfortunately it is deeply disappointing, being marred by serious errors sure to confuse the novice.

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[1] Kuhns, Richard, *Decameron and the Philosophy of Storytelling.* Columbia University Press, New York 2005, p. 6.

[2] Perseus Publishing; Cambridge, MA 2000.

> In order to explain the idea of proof in mathematics, the authors tell the charming tale of how Gauss as a schoolboy is said to have summed the numbers from 1 to 100 by writing the numbers
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> | | | | |
> |---|---|---|---|
> | 1 | 2 | … | 50 |
> | 100 | 99 | … | 51 |
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> and noting that each column adds up to 101. They then show how the same method can be used to sum the numbers from 1 to *n* yielding the formula $n(n+1)/2$ (with the caveat that for *n* odd, 0 must be included). Astonishingly, readers are then told that this proof "is not a proof that the formula holds for *every* positive integer *n*; it's just a proof for any fixed number …" This nonsense is followed by a very brief explanation of mathematical induction as the "usual" way the formula is proved. Next comes a piece of utterly gratuitous misinformation: "There are some philosophers of mathematics who argue that such nonconstructive and/or infinitary principles of inference as mathematical induction should not be admitted into mathematics as a tool of proof." Of course constructivists have no quarrel with mathematical induction.[3]

An even worse example is David Foster Wallace's *Everything and More: A Compact History of* ∞.[4] The amount of confused misinformation is matched by the arrogant manner in which it is presented by this Macarthur award winning author of acclaimed fiction. I quote from my forthcoming review:

> There is a strange lack of the usual markers one would expect to help a reader find his or her way around a book on such a subject: no section or chapter headings, no table of contents, and no index. The early version of the book for reviewers contained not only the usual warning that these were uncorrected proofs, but also the statement that "errors in mathematical and other portions are being corrected." Alas the final version still contains the abundant egregious errors not only in the mathematics, but also in various statements of fact, that make the book painful to read for anyone knowledgeable.[5] The whole is exacerbated by a brash sneering tone inviting the reader to share the author's evident contempt for the niceties of mathematical discourse. Here are some examples:
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> 1. Integration, which is what mathematicians often do when they're stuck on a problem and don't know how to proceed.
> 2. D.E.s [differential equations] can be thought of … as integral calc. on some sort of Class IV hallucinogen.
> 3. Fourier series, which are sort of the sum of two power series, are 3rd or 4th term college math and can be real brainmelters …
> 4. Fourier coefficients, which are so conceptually hairy that we plan to avoid them at almost any cost.
> 5. Unless you don't want just a bunch of abstract math-class vomitus on transfinite set theory.
> 6. Most of us would be inclined to say that $\sqrt{5}$ is an irrational number even if nobody actually proves that it is … The whole issue here is of course incredibly hairy.
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> The niceties of philosophical discourse get their share of abuse. Plato's theory of forms: "… is there an ideal [Platonic] form of … shit?" And, "Zeno … who can actually be seen kicking Socrates' ass … in Plato's *Parmenides.*"[6]

The strategy of using biographies of mathematicians, especially lively anecdotes, to help introduce their discoveries to a general public was pioneered by E.T. Bell. I have

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[3] Notices of the American Mathematical Society, vol. 48(2001) pp.807-813.
[4] Norton, New York 2003.
[5] As I read the book, I made a list of the errors I found; by the end I had counted 86 serious mistakes. I will be pleased to send a copy of the list to any reader who sends me email.
[6] To appear in the Mathematical Intelligencer under the title *David Foster Wallace Mangles the Infinite.*

tried to use this technique in my book on the role of logicians in the origin of all-purpose computers. Here's how I began:

> Situated southeast of the German city of Hanover, the ore-rich veins of the Harz mountain region had been mined since the middle of the tenth century. Because the deeper parts tended to fill with water, they could only be mined so long as pumps kept the water at bay. During the seventeenth century water wheels powered these pumps. Unfortunately, this meant that the lucrative mining operations had to shut down during the cold mountain winter season when the streams were frozen.
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> During the years 1680-1685, the Harz mountain mining managers were in frequent conflict with a most unlikely miner. G.W. Leibniz, then in his middle thirties, was there to introduce windmills as an additional energy source to enable all-season operation of the mines. At this point in his life, Leibniz had already accomplished a lot. Not only had he made major discoveries in mathematics, he had also acquired fame as a jurist, and had written extensively on philosophical and theological issues. He had even undertaken a diplomatic mission to the court of Louis XIV in an attempt to convince the French ``Sun King'' of the advantages of conducting a military campaign in Egypt (instead of against Holland and German territories). Some seventy years earlier, Cervantes had written of the misadventures of a melancholy Spaniard with windmills. Unlike Don Quixote, Leibniz was incurably optimistic. To those who reacted bitterly to the evident misery in the world, Leibniz responded that God, from His omniscient view of all possible worlds, had unerringly created the best that could be constructed, that all the evil elements of our world were balanced by good in an optimal manner. But Leibniz's involvement with the Harz mountain mining project ultimately proved to be a fiasco. In his optimism, he had not foreseen the natural hostility of the expert mining engineers towards a novice proposing to teach them their trade. Nor had he allowed for the inevitable break-in period a novel piece of machinery requires or for the unreliability of the winds. But his most incredible piece of optimism was with respect to what he had imagined he would be able to accomplish with the proceeds he had expected from the project.
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> Leibniz had a vision of amazing scope and grandeur. The notation he had developed for the differential and integral calculus, the notation still used today, made it easy to do complicated calculations with little thought. It was as though the notation did the work. In Leibniz's vision, something similar could be done for the whole scope of human knowledge. He dreamt of an encyclopedic compilation, of a universal artificial mathematical language in which each facet of knowledge could be expressed, of calculational rules which would reveal all the logical interrelationships among these propositions. Finally, he dreamed of machines capable of carrying out calculations, freeing the mind for creative thought. Even with his optimism, Leibniz knew that the task of transforming this dream to reality was not something he could accomplish alone. But he did believe that a small number of capable people working together in a scientific academy could accomplish much of it in a few years. It was to fund such an academy that Leibniz had embarked on his Harz mountain project.[7]

It is a challenge to use the biographical material to carry the reader along while gently introducing technical matters. Although one wants to make use of the various juicy anecdotes that have become part of the culture of mathematics, careful investigation will often reveal such anecdotes to be apocryphal, and the scrupulous author is reduced to saying that the very survival of the anecdote, true or not, is itself revealing. Thus there is no evidence that Poincaré said that some day set theory will be regarded as a disease from which one has recovered although he has often been quoted to this effect.

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[7] *The Universal Computer: The Road from Leibniz to Turing*, W.W. Norton, 2000. Paperback edition: *Engines of Logic: Mathematicians and the Origin of the Computer*, W.W. Norton, 2001.

It is important not to let the heroic stature of the subjects hide their personal defects – in fact some of the most interesting anecdotes arise out of them. A case in point: Gottlob Frege showed almost superhuman honesty and strength of character in his reaction to learning that his just completed two volume master work was deeply flawed. On the other hand, the aftermath of Germany's defeat in World War I found him joining those on the anti-Semitic far right looking for a strong leader. The writer can not avoid complex controversial social and political matters that often elicit strong feelings. For example, I have had a long correspondence with a prominent German scholar who defended Frege from what he insisted was my unfair attack.

Richard Kuhns emphasizes that a story may have a somewhat hidden "latent" content in addition to what lies on its surface.[8] The story I tell traces the circuitous path from Leibniz's original dream to the full-fledged development of the concept of an all-purpose computer by Alan Turing. It tells how Leibniz himself was obliged to do what his patrons deemed important: work on the history of their distinguished family. The latent content that I hope comes across is the importance of permitting, even encouraging, brilliant researchers to follow their own vision of what is important, rather than forcing them to work on what lesser minds see as leading to short-term gains. Who knows what Leibniz might have accomplished had he had the benefit of such enlightened patrons.

[8] Kuhns, *op.cit.*