TRAGIC MATHEMATICS: ROMANTIC IMAGERY AND THE RE-FOUNDING OF MATHEMATICS IN THE EARLY 19TH CENTURY

By Amir Alexander

(DRAFT – Please do not quote)

I. The Mathematical Martyr

Evariste Galois was a young mathematical genius in early 19th century Paris. Despite his groundbreaking solutions to longstanding questions that had long dogged the best mathematicians of his day, Galois did not receive the recognition he deserved from the mathematical establishment. When he submitted an essay to the Royal Academy of Sciences, the paper was promptly lost by none other than Augustin Louis Cauchy – the leading French mathematician of the time. When he submitted a revised version of the memoir to an academy-sponsored competition, all he received was a cold rejection letter from Cauchy's colleague, Poisson, who wrote that he could not make heads or tails of Galois' work.

Disillusioned, Galois turned to radical politics, and soon landed in jail for several months. Immediately upon his release he became entangled in a love triangle over the affections of the mysterious Stephanie, and was challenged to a duel by his rival. Knowing that he may not survive the dawn, Galois spent the night before the duel furiously writing down his latest mathematical insights. "I have no time!" he scribbled in the margins. Tragically his premonition proved true: Galois was shot in the stomach and bled to death on an empty Paris street at the age of 20. His mathematical testament of the previous night, however, survived, and bequeathed to mathematics an entirely new field: Group Theory.[1]

Sadly, the tragic tale of Galois's life and death are not unique in the annals of modern mathematics. It seems, in fact, that mathematics was a very high-risk occupation in the early 19th century. Niels Henrik Abel was a few years Galois's senior, and like him exhibited a remarkable talent for mathematics at an early age. While still in his early twenties he had resolved problems on elliptical functions that had eluded the lifelong efforts of the great Legendre. Backed by a grant from his native Norway, he traveled south to meet and engage the leading mathematicians of the age. Sadly, his reception by the Parisian mathematical establishment was far from friendly. A memoir describing his results that he submitted to Cauchy and Legendre was lost, and he returned to Norway. Poor and discouraged, he died shortly afterwards at the age of 26. An offer for a permanent professorship in the new University of Berlin arrived within days of his death…[2]

Nor were Galois and Abel the only young geniuses to be victimized by a narrow-minded mathematical establishment in the early 19th century. The young Hungarian nobleman Janos Bolyai was a dashing young lieutenant in the service of the Habsburg empire. Influenced by his Father, Farkas Bolyai, who was an accomplished mathematician and longtime friend of the great Gauss, he took on one of the great challenges of classical geometry: the proof of Euclid's parallel postulate.

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[1]) Based on the account by E.T. Bell.
[2]) Based on Bell.

His father, who had worked on the question years before, was not pleased:

> **You should not tempt the parallels in this way, I know the way until its end – I also have measured this bottomless night. I have lost in it every light, every joy of my life . . . You should shy away from it as though from a lewd intercourse, it can deprive you of all your health, your leisure, your peace of mind, and your entire happiness. . . This infinite darkness might perhaps absorb a thousand giant Newtonian towers, it will never be light on Earth, and the miserable human race will never have something absolutely pure, not even geometry . . . (F. Bolyai to J. Bolyai, 1820).**

The young Bolyai, however, was irresistibly drawn to the classic problem of the parallels. By 1823 he had developed an alternative geometrical system, which became known as non-Euclidean geometry. Impressed despite his earlier warnings, the elder Bolyai sent his son's treatise to his old friend Gauss. The response was a startling blow to young Bolyai: "I am unable to praise this work," wrote The Prince of Mathematicians, because "to praise it would be to praise myself." He himself, Gauss said, had already discovered all that is contained in the young Bolyai's manuscript decades before. Janos grew angry and discouraged by the attitude of the elder mathematician. He retired to the family estate and published no more mathematical work for the rest of his life.

The persistence of this "tragic" tradition is evident in the stories of later generation mathematicians as well. Georg Cantor was denied the recognition he deserved for his discovery of transfinite numbers by the enmity of powerful Kronecker. He spent his days teaching at a provincial university, suffering repeated mental breakdowns, and ended his life in an insane asylum. Srinivasa Ramanujan, an Indian genius with an uncanny intuition for numbers, spent several years in Cambridge in the company of G. H. Hardy, but died penniless and alone in India in 1920. Kurt Gödel, who in his youth shook the foundations of mathematics with his “Incompleteness Theorem,” cut a sad figure as a delusional old man at the Institute for Advanced Studies in Princeton years later. Most recently the portrayal of mathematician John Nash in the movie *A Beautiful Mind* presents the same popular image of the mathematician as a mentally fragile genius. In the world of fiction (as against the “fictionalized”) I would mention “Petros Papachristos,” the fictional mathematician in Apostolos Doxiadis’s *Uncle Petros and Goldbach’s Conjecture*, who wasted his life in an effort to prove an elusive theorem, is the very embodiment of the tragic modern mathematician.[3]

I cannot vouch for the historical accuracy of these familiar biographies, which were disseminated most successfully in E.T. Bell’s *Men of Mathematics*, first published in 1937. Most likely they contain a kernel of truth, modified to fit a tragic romantic mold. In the case of Galois – by far the most dramatic of the collection - modern scholarship has cast extensive doubt on many aspects of the story. Galois’s own erratic and paranoid behavior, rather than the enmity of a faceless “establishment,” was likely responsible for a good portion of his troubles. But that is not the point: what is significant about Galois’s story is that it quickly gained widespread currency as an emblematic tale of mathematical genius shunned by an uncaring world.

[3]) Apostolos Doxiadis, *Uncle Petros and Goldbach’s Conjecture*

In some ways Galois can be considered the author of his own legend. A few months before his death, he wrote a scathing "preface" to his work, where he railed against the mathematical giants of his time, and drew parallels between his own fate and Abel's a few years before.

> **If I had to address anything to the great of the world or the great in science . . . I swear it would not be in thanks. I owe to the ones that I have published the first of these two memoirs so late, to the others that I have written it all in prison, which it would be wrong to consider a place of meditation . . . It is not my subject to say how and why I was detained in prison, but I must say how my manuscripts have been lost most often in the cartons of Messieurs the members of the Institute, although in truth I cannot imagine such thoughtlessness on the part of men who have the death of Abel on their consciences.**
>
> (Galois, 1832)

In Galois's own mind, he was a victim of deliberate persecution by the "great," who willfully refuse to acknowledge the brilliance of his accomplishment, As we have seen, the accepted biographies of many other mathematicians follow this tragic pattern closely.

Whatever the precise biographical details in each case, we are, overall, presented with a standard tale of the life of a mathematical genius. The young prodigy, the story goes, shows a remarkable mathematical aptitude from an early age and soon overtakes the leading mathematicians of his time. Confident in his abilities, the genius presents his groundbreaking work to his seniors, expecting that it will be received with the admiration it deserves. Sadly, however, the established mathematicians, comfortable in their institutional chairs, refuse to acknowledge the gift of their young colleague. Through ignorance or sheer wickedness they reject the genius's masterpiece, leaving him crushed and disillusioned. Tragic consequences soon follow.

Significantly, in this story, worldly success is viewed with profound suspicion. Professionally successful mathematicians, instead of using their positions to advance the prospects of brilliant successors, seem inclined to preserve their station by suppressing young minds. Cauchy is accused of "losing" manuscripts of revolutionary significance not once, but twice! Gauss, in his old age, was more concerned with preserving his priority claims for work he never bothered to publish than with promoting new mathematical thought, and Kronecker used his position as editor of the leading mathematical journal to prevent the publication of Cantor's revolutionary work. In contrast, worldly failure, in the form of early death, disgrace, disillusionment, or even madness, appears to be a good indicator of profound mathematical insight.

The most striking aspect of this story is that it runs counter to the traditional image of the mathematician as it existed at the time. The leading 18th century mathematicians, who preceded the generation of Galois and Abel, showed no tragic inclinations whatsoever. In fact, they tended to be powerful public figures highly placed within the intellectual establishment of their time. The mathematical Bernoulli clan of Basel, for example, was so successful, that they installed family members in the most prestigious mathematical chairs in Europe for three generations(!). Jean LeRond d'Alembert was a leading "philosophe," a stalwart of the fashionable salons, lifelong member of the Royal Academy of Sciences, and perhaps most significantly – co-editor of the *Encyclopedie*. Pierre-Louis Moreau de Maupertuis was the heroic leader of a geographical expedition and later president of the Berlin Academy and friend of Frederick II of Prussia. And Leonhard Euler, perhaps the greatest and certainly the most prolific of 18th Century mathematicians, held mathematics chairs in St. Petersburg, Berlin, and again St. Petersburg, and was a personal acquaintance and correspondent of Kings, Emperors, and Princes.

Not only were the leading mathematicians of the Enlightenment successful men of affairs, but rejection by the mathematical establishment conferred no mark of distinction on the victim. When the Wolffian mathematician Samuel Koenig debated Maupertuis over the "Principle of Least Action" in the 1750s, he was effectively silenced and consigned to obscurity. [4] "His book is buried with him, if it ever existed" wrote academician Jean-Bertrand Merian to Maupertuis with satisfaction when Koenig died soon after. And that has remained pretty much the judgement of history to this day.[5] I am, in fact, unable to think of a single 18th century mathematician whose work went unacknowledged at the time, but who gained fame posthumously. The verdict of the leading contemporary mathematicians was and remains final.

[4]) Mary Terrall, *The Man who Flattened the Earth*, 292-309
[5]) Mary Terrall, *The Man who Flattened the Earth*, 355.

The end of the 18^{th} and the beginning of the 19^{th} century was clearly a watershed in the general understanding of the role and character of the mathematician. On the one side of the divide were some of the "Great Men" of the Enlightenment: public figures, writers and philosophers, leading members of fashionable society as well as of the Republic of Letters, courtiers to emperors and kings. On the other are lonely geniuses toiling in obscurity, whose brilliance and insight go unacknowledged by an arrogant establishment and an uncaring world. On the one side are Maupertuis and Euler; on the other, Galois and Abel.[6]

Needless to say, the story of frustrated young genius, is certainly not unique to mathematics. It is, rather, a staple of 19^{th} century romanticism, and its heroes range from musicians to poets to painters. The young Mozart comes to mind, who - despite his remarkable childhood promise – never received the rewards due to his talents at the court of Emperor Joseph II. The fate of the historical Mozart was sad enough - he was constantly in debt and died penniless at the age of 35; the mythical Mozart, in true tragic fashion, was doomed by the envy of the mediocre but powerful court composer Antonio Salieri. The parallels with Galois's shabby treatment by Cauchy and the French Academy, and its tragic consequences, are striking.

In similar fashion the young Lord Byron was driven from England by hypocritical London society. He was to die a short while later at the age of 36 while engaged in a new

[6]) I do not mean to suggest that all practicing mathematicians in the 19th Century were, in fact, alienated loners . Certainly there were successful 19th Century "establishment mathematicians," such as Augustin-Louis Cauchy and Karl Weierstrass. But the prevailing image of what it means to be a mathematician had undergone a profound change. A new "type" or mathematician – unknown in the 18th century –emerged in the early 19th century, and its echoes dominate popular views of the field to this day.

romantic errand - helping the Greeks win their independence from the Ottoman Empire. Vincent Van Gogh, though a member of a later generation, is probably the ultimate tragic genius, who never sold a single painting during his lifetime and committed suicide at age 37. Each of these, and no doubt many others, embody the romantic myth, as do Galois and Abel.

Now in music, poetry, and painting, the advent of the tragic romantic hero undoubtedly went hand in hand with a profound shift in the style and practices of these fields. Romantic music was expressive of extreme emotion like nothing that had preceded it. Much the same can be said of romantic poetry, which emphasized emotional expressiveness over stylistic elegance. In painting, there is no question that Van Gogh's generation saw a radical revolution in the meaning and purpose, as well as the practice of the art.

Mathematics appears at first glance to be the exception. Unlike the arts, which are perpetually shaped and reshaped by trends and fashions, mathematics is bound to its own unyielding internal logic. The dark and impassioned operas of Wagner may be worlds away from the esthetically pleasing concertos of Bach; but the series expansion of a function, or the shape of a curve, do not change, no matter whether their discoverer was a successful academician or a poverty-stricken outcast. Their truth is determined by the fixed and unchanging logic of mathematical reasoning, which is always, and everywhere, the same. Because of this, mathematics of all fields appears to be the most insulated from the effects of transitory culture. The popular stories told about mathematicians may vary in different times and contexts; but the actual content of mathematics, does not.

Nevertheless, I would like to argue otherwise. Mathematical stories and mathematical practice are indeed linked, and profoundly so. The new biographical narrative of mathematicians in the early 19th century went hand in hand with a shift in the interests and approaches of practicing mathematicians. And this was no coincidence.

II. The Re-Birth of Mathematics

The early 19th century is sometimes referred to as the period of the "re-birth" of mathematics. There is good reason for this: a new insistence on logical rigor and internal consistency pervaded the field, surpassing anything that had gone before. 18th century mathematicians were usually content to reach correct results. Any methodological difficulties along the way were obviated by the fact that a true result was ultimately reached.

But to their 19th century successors this approach seemed dolefully inadequate. For them, mathematics must be internally self-consistent, rigorous, and established on secure foundations. Only then could it be "applied" to other fields. A practice that relied for its truth value on physical reality or technological prowess was no mathematics at all. Mathematical techniques, such as the calculus, whose foundations were suspect, may be useful tools in the near term, but were bound to lead to error in the long run if the basis of their effectiveness was not clarified.

For a taste of the transformation of mathematical attitudes, consider the contrast between the mathematical views of Joseph Louis Lagrange, one of the giants of 18^{th} century mathematics, and those of Abel, one of the tragic heroes of the 19^{th} century. “It seems to me that the mine has maybe already become too deep, and unless one finds new veins it might have to be abandoned,” Lagrange writes in an 1871 letter to d’Alembert. “Physics and chemistry now offer a much more glowing richness and much easier exploitation.” For Lagrange, as for many contemporaries, mathematics had proven so successful and effective in resolving problems, that it may have little room for improvement.

But where Lagrange saw a field that was so well understood that it was approaching completion, Abel saw only darkness and obscurity:

> **I will apply all my strength to bringing light to the great darkness that unquestionably exists in analysis. It totally lacks any plan and system, so it is really very strange that it is studied by so many and worst of all, that it is not treated rigorously at all. There are very few theorems in the higher analysis which have been proved with convincing rigor. . . it is very strange that after such a procedure there exist only few of the so-called paradoxes.**
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> (Abel, letter to Hantseen, 1826)

The correct results, which for Lagrange were both the purpose of Analysis and the ultimate guarantee of its viability, were for Abel merely a puzzling aberration. For Abel, a mathematical discipline that is not systematic and rigorous is hardly worthy of its name.

The main concern of 19th century mathematicians was not finding useful new results, but systematizing and developing the internal structure of mathematics itself. Since this has largely remained the concern of professional mathematicians to this day, it is no wonder that the early 19th century is often viewed as the time of the birth of modern mathematics.

The novelty of this approach can best be appreciated by looking at some examples of the problems that occupied the leading mathematicians of the previous century. The "discovery" of the calculus by Newton and Leibniz in the late 17th century set the stage for a century of work exploring the power and possibilities of the new techniques. "Analysis," as Enlightenment mathematicians liked to call the calculus and its allied fields, was originally developed to deal with the physical realities of space and the problem of motion. It was therefore only natural for 18th century mathematicians to focus almost exclusively on questions closely associated with the physical world.[7]

One type of problem that was the focus of much attention beginning in the late 17th century was the determination of curves formed by mechanical action or forces. The catenary was the shape of a curve formed by a hanging chain; the brachistachrone is the shortest path followed by an object sliding from one point to another – not on the same vertical line – in the least possible time; the tautochrone is a curve along which a body

[7]) Thomas L. Hankins, *Science and the Enlightenment*, 21 ff.

will arrive at a given final point in the same amount of time (under the influence of gravity), no matter where on the curve it began its slide, and so on. These questions and other like them occupied the Bernoulli brothers in the early decades of the 18th century.[8] The correct mathematical description of the motions of a vibrating string was the subject of one of the great mathematical controversies of the century, with d'Alembert and Euler engaged on opposite sides of the issue. And Maupertuis, another of the leading "geometers" of the age, gained his mathematical reputation largely formulating the "Principle of Least Action" to describe the inner logic of the operations of nature.[9]

The movement in analysis throughout the 18th century went from the geometrical and particular to the algebraic and general. Starting with the investigation of specific geometrical curves like the catenary (Johann and Jacob Bernoulli), the focus soon shifted to more general questions of the behavior of physical and geometrical bodies (d'Alembert – dynamics, Maupertuis – least action). Finally, in the work of Euler and Lagrange, 18th century analysis became the study of the interrelationships of fully abstract and general algebraic functions. But even in its most abstract forms, 18th century analysis remained fundamentally about the structure of the physical world. On the one hand, algebraic analysis was rooted in the investigation of particular geometrical and physical objects; on the other hand, the relationships between these algebraic functions revealed hidden truths about these same physical and geometrical objects. To 18th century practitioners, in other words, the world was seen as fundamentally mathematical, and mathematical investigations were therefore fundamentally about the world.

[8]) Thomas L. Hankins, *Science and the Enlightenment*, 25 – 28.
[9]) Mary Terrall, *The Man who Flattened the Earth*

The dual relationship between mathematics and the physical world was elucidated in one of the Enlightenment's foundational texts – d'Alembert's "Preliminary Discourse" to the *Encyclopedie* (1751). Algebra, d'Alembert explains, is nothing but the most general relationships that pertain between physical objects, once their specific physical attributes (impenetrability, force) are removed. It is reached by the systematic abstraction of the general characteristics of physical objects. He then continues:

> **This science (Algebra) is the farthest outpost to which the contemplation of the properties of matter can lead us, and we would not be able to go further without leaving the material universe altogether. But such is the progress of the mind in its investigations, that after that after having generalized its perceptions to the point where it can no longer break them up further into their constituent elements, it retraces its steps, reconstitutes anew its perceptions themselves, and, little by little and by degrees, produces from them the concrete beings that are the immediate and direct objects of our sensations . . . Mathematical abstractions help us in gaining this knowledge, but they are useful only insofar as we do not limit ourselves to them.**

That is why having so to speak exhausted the properties of shaped extension through geometrical speculation, we begin by restoring to it impenetrability which constitutes physical body and was the last sensible quality of which we had divested it. The restoration of impenetrability brings with it the consideration of the actions

of bodies on one another, for bodies act only insofar as they are impenetrable. It is theme that the laws of equilibrium and movement, which are the object of Mechanics, are deduced. We extend our investigations even to the movement of bodies animated by unknown driving forces or causes, provided the law whereby these causes act is known or supposed to be known.[10]

For d'Alembert then, Mathematics is nothing but physical reality shorn of its sensible properties; conversely, the physical world is simply mathematical abstraction once sensible physical properties are restored to it.

Whether or not most 18th century mathematicians subscribed to d'Alembert's ontological views, their actual mathematical practices closely followed their outlines. Beginning early on with questions derived directly from physical reality, mathematicians gradually moved towards increasing abstraction and generalization. But even at its furthest algebraic outpost, manifested in the work Euler and Lagrange, the subject matter of mathematics remained physical reality. Not only were the problems of mathematics ultimately drawn from the "real world," but physical reality also guaranteed the validity of mathematical truths. This is made very clear in 18th century attitudes towards the calculus.

Ever since infinitesimal methods were introduced into mainstream mathematics around 1600, mathematicians had been aware of the logical pitfalls they entail. These were

[10]) Jean Le Rond d'Alembert, *Preliminary Discourse to the Encyclopedia of Diderot*, Richard N. Schwab trans. (Chicago: University of Chicago Press, 1995), 20-21.

founded on problems well-known since antiquity – the problem of incommensurability and the paradoxes of Zeno. Bishop George Berkeley, who famously criticized Newton's "evanescent increments" as "the ghosts of departed quantities" in *The Analyst* of 1734, was only a recent and exceptionally witty critic of infinitesimals.[11] But despite the foundational vulnerability of the calculus, most 18th century practitioners showed remarkably little concern for the effectiveness and consistency of their method. The technique had to be fundamentally correct, they reasoned, or it would not so effectively describe the real world. Effectiveness was therefore an unmistakable indication of the fundamental legitimacy of the method, and "reality" was not only a source of inspiration, but a guarantor of mathematical truth.

Things could not have been more different in the following century. Quite suddenly, many mathematicians seemed to lose interest in the physical roots and manifestations of their science. Mathematics is now seen as a science unto itself, whose value can only be judged by its own internal standards. Now, it seemed, mathematics could only be worthy of its name if it was rigorous, self-consistent, and systematic. Effectiveness in problem solving is certainly praiseworthy, but it cannot endow a given approach with mathematical legitimacy. This can only be accomplished by the systematic exposition of a subject through rigorous deduction from secure foundations.

Some of the most celebrated mathematical work of this period involved a reinterpretation of the accomplishments of the preceding century. Most famously, Cauchy and Bolzano established the calculus as a self-contained and rigorous deductive system. Whereas their

[11]) Bishop George Berkeley, *The Analyst*.

predecessors relied on the undeniable success of analysis in describing the physical world to legitimize their use of this problematic approach, Cauchy and Bolzano claimed to rely solely on the internal consistency of their own mathematical system. They re-defined fundamental concepts such as the "limit" using less intuitive but more internally consistent terms, and based their interpretation of the calculus on the concept of the "derivative" instead of the problematic "differential." Their efforts did little to improve the effectiveness of analysis as a tool for investigating the physical world; it was, nevertheless, the foundation of modern mathematical analysis as it is practiced to this day.

In other cases, 19th century mathematicians invented entire new fields, which were unthinkable to the older generation. Non-Euclidean Geometry is a case in point. In the 18th century several noted mathematicians took it upon themselves to prove Euclid's problematic 5th postulate, which states that given a straight line and a point not on it, only one parallel to the given line passes through the point. By assuming the proposition false, Girolamo Saccheri, Johann Heinrich Lambert, and Adrien-Marie Legendre, set about looking for a contradiction that would prove that the postulate is necessarily true. The quest proved surprisingly difficult, and each in turn had to bend their standards considerably in order to prove to their own (incomplete) satisfaction that the 5th postulate is indispensable to the consistency of geometry. None of them, nevertheless, doubted that the proof could and would ultimately be found. Since geometry was ultimately about the world, and since the 5th postulate correctly described our worldly experience, there was no question that it was ultimately a necessary part of geometry.

19th century mathematicians took a very different approach. Beginning with Gauss (who never published his results) and continuing in the work of Janos Bolyai and Nikolai Lobatchevski, the new mathematicians felt free to develop an alternative geometry that had no correlate in the physical world. The new non-Euclidean geometry described a world in which not one, but an infinite number of parallels to a given line pass through a point not on it. In this strange world the sum of the angles of a triangle was less than two right angles, and it was – furthermore dependent on the area of the triangle. The greater the area, the smaller the sum of the angles. Similar triangles did not exist, and scaling was therefore impossible.

To 18th century mathematicians, this non-Euclidean world was at best irrelevant, or at worst – a deliberate fabrication. But to Bolyai and Lobatchevski it described a world just as mathematically legitimate as the familiar Euclidean one. In fact, the inapplicability of the new geometry to the physical world was precisely what made it interesting: it made the point in the clearest fashion that mathematics could only be judged by its own internal standards. Any alternative geometry was just as "real" as Euclid's, as long as it was systematic and internally consistent. Like Cauchy and Bolzano in the case of calculus, the pioneers of non-Euclidean geometry insisted that the legitimacy of a mathematical system lies entirely in its own coherence and self consistency.

A similar trend can be seen in young Galois' work on what became group theory. Galois' starting point was in the work of Lagrange, who wrote a treatise on 5th degree equations and why they are not solvable by radicals (i.e. by a standard algebraic formula). But whereas Lagrange had focused on the specific mathematical problem with a view to its solution, Galois developed an abstract and general mathematical, which gave one deep insight into the nature of algebraic equations but was completely useless for practical resolution of equations. Explaining his approach he wrote:

> **If you now give me an equation that you have chosen at your pleasure, and if you want to know if it is or is not solvable by radicals, I could do nothing more than indicate to you the means of answering you're your question, without having to give myself or anyone the task of doing it. In a word, the calculations are impractical. . .**
>
> **All that makes this theory beautiful, and at the same time difficult, is that one has always to indicate the course of analysis and to foresee its results without ever being able to perform [the calculations].**

For anyone trying to resolve a particular equation, in other words, the theory is worthless, as Galois readily admits. In fact, the beauty of the method derives precisely from its impracticability. Like Cauchy's new foundations for analysis, and Bolyai's non-Euclidean geometry, Galois' theory was concerned with the internal coherence of a mathematical system, not with the solution of specific problems based in physical reality.

In essence, whereas the great 18th century masters saw mathematics as inseparable from the physical world, 19th century mathematicians radically divorced mathematics from the world. Mathematics now became its own separate universe – perfect, logical, consistent, and beautiful – and very different from the imperfect unpredictable universe we see around us. Whereas our own world is governed by the unyielding realities of physical nature and the contingencies of human existence, the mathematical world knows no such limitations. Its truths are eternal, unchanging, and perfect, regardless of any manifestation they may or may not have in the physical world, and they exist on a different plane of reality than anything we see around us. “There exist realities other than sensible objects” wrote Cauchy, in his introduction to the *Cours d’Analyse*, his classic text which reestablished the foundations of the calculus.[12] He, and others of his generation, followed this credo to the letter.[13]

Is it a wonder then that the mathematics of the 19th century required a very different practitioner than the mathematics of earlier generations? As long as mathematics was part of the physical world, it was only natural to expect that a practicing mathematician would be part of this “real” world as well. 18th century mathematicians studied the physical world intensely, based their mathematical knowledge on their understanding of the world, and their physical theories on their understanding of mathematics. What they studied was

[12]) Cauchy, *Cours d’Analyse*.

[13]) While the sharp separation between mathematics and the physical world was the dominant trend in 19th century academic mathematics, a rival tradition which viewed mathematics as fundamentally about the physical world continued alongside it. Its leading member included Jean-Baptiste Joseph early in the century, and Henri Poincare at its close. The rise of statistics is also associated with this tradition, which continued uninterrupted from the 18th century.

Mention that the new approach was not the only one – Fourier, Poincare, Comte, Statistics,etc. – all of which kept the 18th century tradition alive. But mainstream professional mathematics was nevertheless transformed , and dominated by the new approach.

essentially accessible to all of us through our senses, our experiences, our histories. They were literally "men of the world" – intellectually, professionally, and personally. Men who immersed themselves in the study of reality could be expected to feel at home in the world. Their long and prosperous careers are sure testimony that d'Alembert, Euler, and their colleagues were indeed very comfortable in our mundane universe.

Things were very different, however, in the 19th century, when mathematics existed in a universe separate from our own, with its own rules and its own strange realities. Mathematicians now were not those with a special and deep understanding of our own world, but those unaccountably gifted with privileged access to an alternative and higher reality. Is it a wonder that those who were able to glimpse the dazzling beauty of the mathematical universe would find life on Earth burdensome and confusing? Hardly. Galois and Abel, unlike d'Alembert and Euler, lived their true lives in another world, accessible only to those gifted with the mathematical site. They did not belong in our own physical universe, with its contingent realities, politics, and power structures. They were creatures of a higher and better universe, that most mortals never glimpse. The fact that they had to live their physical lives in the mundane circumstances of 19th century Europe was, simply put, a tragedy. Unsurprisingly, it ended with disillusionment and an early death.

The young Janos Bolyai is perhaps most eloquent in expressing the ecstatic quality of the new mathematics when in 1823 he wrote breathlessly to his father about his work in non-Euclidean geometry:

> **I have made such wonderful discoveries that I have been almost overwhelmed by them . . . I can only say this: *I have created a new universe from nothing*.**

(J. Bolyai to F. Bolyai, 1823)

Indeed he had. But as he and other mathematical contemporaries were to discover, the personal price of such a monumental achievement could be steep.

The romantic tragedy of mathematics emerged at precisely the same time as the re-founding of mathematics in the early 19th century. This is no coincidence: A new type of mathematical practice went hand in hand with a new story about the life of mathematicians and the meaning of the field itself.